Global Types with Internal Delegation and Connecting Communications

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Alice Cat Bank Example

Alice Cat Bank Example

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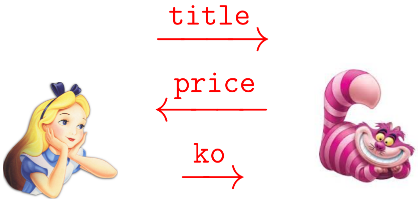
Alice Cat Bank Example



Alice Cat Bank Example

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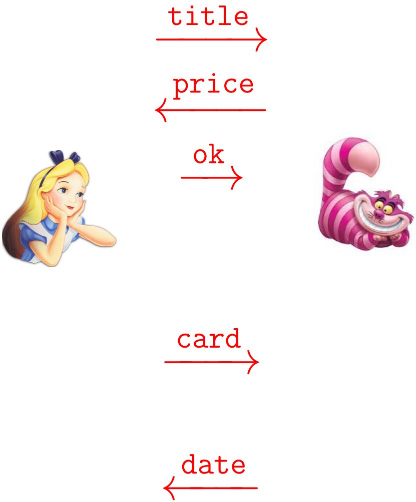
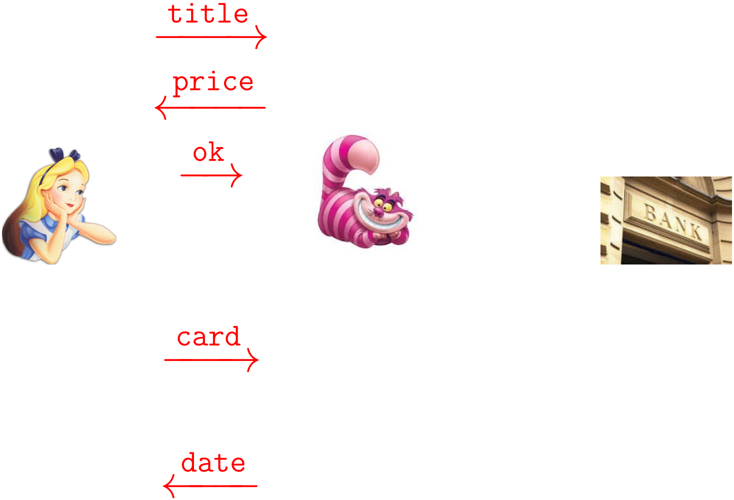
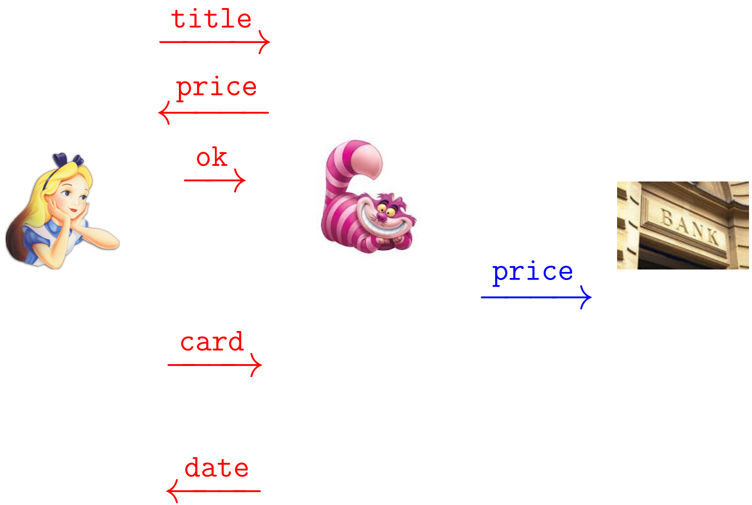
Alice Cat Bank Example



Alice Cat Bank Example



Alice Cat Bank Example



Alice Cat Bank Example

title →

price ←

ok →

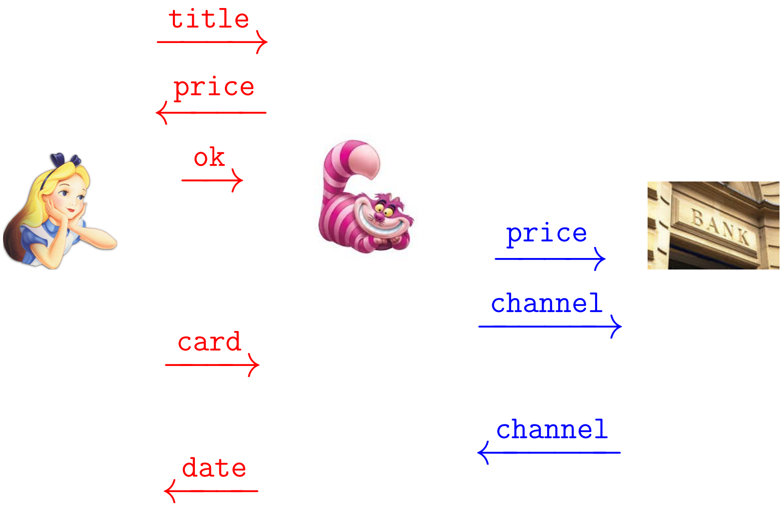
price →

channel →

card →

date ←

Alice Cat Bank Example



Two Global Types

G_{ac}

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\ (\ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\ )$$
$$\boxplus$$
$$\mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End}$$
$$)$$

G_{cb}

$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{B};$$
$$\mathtt{C} \xrightarrow{\mathsf{T}} \mathtt{B};$$
$$\mathtt{B} \xrightarrow{\mathsf{T}'} \mathtt{C}; \mathsf{End}$$

$\mathsf{T} = \mathtt{A}?\,\mathtt{card}\,;\mathsf{T}'$ $\quad$ $\mathsf{T}' = \mathtt{A}!\,\mathtt{date}\,;\mathsf{End}$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$ *connecting communication*

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$
$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\ \textit{forward delegation}$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C}; \\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A}; \\
\qquad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C}; \\
\qquad\quad\; \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B}; \\
\qquad\quad\; \mathtt{C} \circ \langle\!\langle \bullet \mathtt{B}; \\
\qquad\quad\; \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};
\end{array}
$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$
$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}; \quad \textit{backward delegation}$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad \color{red}{\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad \color{blue}{\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}
\end{array}
$$

One Global Type

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$
$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}$$
$$\boxplus$$

One Global Type

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad\Big(\ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}\\
\qquad \boxplus\\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\ \mathsf{End}\ \ \Big)
\end{array}
$$

Start with **Forward delegation**

$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}$

$\circ\langle\!\langle\bullet\mathtt{B}$ $\mathtt{C}\circ\langle\!\langle\bullet$

Terminology (active/passive):

- **active forward delegation** $\circ\langle\!\langle\bullet\mathtt{B}$
- **passive forward delegation** $\mathtt{C}\circ\langle\!\langle\bullet$.

Message sent to Cat goes directly to Bank

$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$

$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}$

$\mathtt{C}!\,\mathtt{card}\,;$

$\circ\langle\!\langle\bullet\mathtt{B}$

$\mathtt{C}\circ\langle\!\langle\bullet;$
$\mathtt{A}?\,\mathtt{card}$

Trust assumption: Cat does not have authority to handle card.

End with **backward delegation**

$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$$

$\mathtt{C}!\,\mathtt{card}\,;$

$\circ\langle\!\langle\bullet\mathtt{B};\mathtt{B}\bullet\rangle\!\rangle\circ$

$\mathtt{C}\circ\langle\!\langle\bullet;$
$\mathtt{A}?\,\mathtt{card}\,;$
$\bullet\rangle\!\rangle\circ\mathtt{C}$

Terminology (active/passive):

- **active backward delegation** $\bullet\rangle\!\rangle\circ\mathtt{C}$
- **passive backward delegation** $\mathtt{B}\bullet\rangle\!\rangle\circ$.

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

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$$P \quad ::= \quad \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \quad \textit{external choices of inputs}$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$P \quad ::= \quad \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \ \textit{internal choices of outputs}$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$P ::= \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I} \mathsf{p}_i!\Lambda_i; P_i$$
$$| \quad \mathsf{p}\circ\langle\langle\bullet; P \textit{ forward delegation with principal}$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{lll}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \text{ \textit{forward delegation with deputy}}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I}\, \mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \\
 & | & \bullet\rangle\!\rangle\circ\mathsf{q}; P \ \textit{backward delegation with principal}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcllll}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i & \mid & \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i & \\
& \mid & \mathsf{p}\circ\langle\!\langle\bullet; P & \mid & \circ\langle\!\langle\bullet\mathsf{p}; P & \\
& \mid & \bullet\rangle\!\rangle\circ\mathsf{q}; P & \mid & \mathsf{q}\bullet\rangle\!\rangle\circ; P & \textit{backward delegation with deputy}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \\
 & | & \bullet\rangle\!\rangle\circ\mathsf{q}; P \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ; P \\
 & | & \mu X.P \quad | \quad X \quad | \quad \mathbf{0}
\end{array}
$$

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \\
 & | & \bullet\rangle\!\rangle\circ\mathsf{q}; P \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ; P \\
 & | & \mu X.P \quad | \quad X \quad | \quad \mathbf{0}
\end{array}
$$

internal and external choices must not be ambiguous

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i \quad \mid \quad \bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i;P_i \\
& \mid & \mathsf{p}\circ\langle\!\langle\bullet;P \quad \mid \quad \circ\langle\!\langle\bullet\mathsf{p};P \\
& \mid & \bullet\rangle\!\rangle\circ\mathsf{q};P \quad \mid \quad \mathsf{q}\bullet\rangle\!\rangle\circ;P \\
& \mid & \mu X.P \quad \mid \quad X \quad \mid \quad \mathbf{0}
\end{array}
$$

internal and external choices must not be ambiguous

Processes

Λ ranges over λ and $\overset{\lambda}{\leftrightarrow}$

$$
\begin{array}{rcl}
P & ::= & \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \quad | \quad \bigoplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \\
& | & \mathsf{p}\circ\langle\!\langle\bullet; P \quad | \quad \circ\langle\!\langle\bullet\mathsf{p}; P \\
& | & \bullet\rangle\!\rangle\circ\mathsf{q}; P \quad | \quad \mathsf{q}\bullet\rangle\!\rangle\circ; P \\
& | & \mu X.P \quad | \quad X \quad | \quad \mathbf{0}
\end{array}
$$

internal and external choices must not be ambiguous

$$\mathtt{A}?\mathtt{title}\,;\,\mathtt{A}!\mathtt{price}\,;\,(\mathtt{A}?\mathtt{ok}\,;\,\mathtt{B}!\overset{\mathtt{price}}{\leftrightarrow}\,;\,\circ\langle\!\langle\bullet\mathsf{B};\,\mathsf{B}\bullet\rangle\!\rangle\circ;\,\mathtt{A}!\mathtt{date}\,+\,\mathtt{A}?\mathtt{ko}\,)$$

Networks

$$\mathtt{A}[\![\, \mathtt{C!\,card} \,;\, \mathtt{C?\,date} \,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet \mathtt{B};\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A!\,date} \,]\!] \parallel$$
$$\mathtt{B}[\![\, \mathtt{C}\circ\langle\!\langle \bullet;\, \mathtt{A?\,card} \,;\, \bullet \rangle\!\rangle \circ \mathtt{C} \,]\!]$$

Networks

$$\mathtt{A}[\![\, \mathtt{C!\,card\,;\,C?\,date} \,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet \mathtt{B};\, \mathtt{B}\bullet \rangle\!\rangle\circ;\, \mathtt{A!\,date} \,]\!] \parallel$$
$$\mathtt{B}[\![\, \mathtt{C}\circ\langle\!\langle \bullet;\, \mathtt{A?\,card}\,;\, \bullet\rangle\!\rangle\circ\mathtt{C} \,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C!\,card\,;\,C?\,date} \,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet \rangle\!\rangle\circ;\, \mathtt{A!\,date} \,]\!] \parallel \mathtt{C}[\![\, \mathtt{A?\,card}\,;\, \bullet\rangle\!\rangle\circ\mathtt{C} \,]\!]$$

Networks

$$\mathtt{A}[\![\, \mathtt{C!\,card\,;\,C?\,date} \,]\!] \parallel \mathtt{C}[\![\, \circ\langle\!\langle \bullet \mathtt{B};\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A!\,date} \,]\!] \parallel$$
$$\mathtt{B}[\![\, \mathtt{C}\circ\langle\!\langle \bullet;\, \mathtt{A?\,card}\,;\, \bullet\rangle\!\rangle\circ\mathtt{C} \,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C!\,card\,;\,C?\,date} \,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A!\,date} \,]\!] \parallel \mathtt{C}[\![\, \mathtt{A?\,card}\,;\, \bullet\rangle\!\rangle\circ\mathtt{C} \,]\!]$$
$$\Downarrow$$
$$\mathtt{A}[\![\, \mathtt{C?\,date} \,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet \rangle\!\rangle \circ;\, \mathtt{A!\,date} \,]\!] \parallel \mathtt{C}[\![\, \bullet\rangle\!\rangle\circ\mathtt{C} \,]\!]$$

Networks

$$\texttt{A}[\![\, \texttt{C}!\,\texttt{card}\,;\texttt{C}?\,\texttt{date}\,]\!] \parallel \texttt{C}[\![\, \circ\langle\!\langle \bullet\texttt{B};\, \texttt{B}\bullet \rangle\!\rangle\circ;\, \texttt{A}!\,\texttt{date}\,]\!] \parallel$$
$$\texttt{B}[\![\, \texttt{C}\circ\langle\!\langle \bullet;\, \texttt{A}?\,\texttt{card}\,;\, \bullet\rangle\!\rangle\circ\texttt{C}\,]\!]$$
$$\Downarrow$$
$$\texttt{A}[\![\, \texttt{C}!\,\texttt{card}\,;\texttt{C}?\,\texttt{date}\,]\!] \parallel \overset{*}{\texttt{C}}[\![\, \texttt{B}\bullet\rangle\!\rangle\circ;\, \texttt{A}!\,\texttt{date}\,]\!] \parallel \texttt{C}[\![\, \texttt{A}?\,\texttt{card}\,;\, \bullet\rangle\!\rangle\circ\texttt{C}\,]\!]$$
$$\Downarrow$$
$$\texttt{A}[\![\, \texttt{C}?\,\texttt{date}\,]\!] \parallel \overset{*}{\texttt{C}}[\![\, \texttt{B}\bullet\rangle\!\rangle\circ;\, \texttt{A}!\,\texttt{date}\,]\!] \parallel \texttt{C}[\![\, \bullet\rangle\!\rangle\circ\texttt{C}\,]\!]$$
$$\Downarrow$$
$$\texttt{A}[\![\, \texttt{C}?\,\texttt{date}\,]\!] \parallel \texttt{C}[\![\, \texttt{A}!\,\texttt{date}\,]\!] \parallel \texttt{B}[\![\, \mathbf{0}\,]\!]$$

Networks

$$\texttt{A}\llbracket\, \texttt{C}!\,\texttt{card}\,;\texttt{C}?\,\texttt{date}\,\rrbracket \parallel \texttt{C}\llbracket\, \circ\langle\!\langle \bullet\texttt{B};\, \texttt{B}\bullet\rangle\!\rangle\circ;\, \texttt{A}!\,\texttt{date}\,\rrbracket \parallel$$
$$\texttt{B}\llbracket\, \texttt{C}\circ\langle\!\langle\bullet;\, \texttt{A}?\,\texttt{card}\,;\, \bullet\rangle\!\rangle\circ\texttt{C}\,\rrbracket$$
$$\Downarrow$$
$$\texttt{A}\llbracket\, \texttt{C}!\,\texttt{card}\,;\texttt{C}?\,\texttt{date}\,\rrbracket \parallel \overset{*}{\texttt{C}}\llbracket\, \texttt{B}\bullet\rangle\!\rangle\circ;\, \texttt{A}!\,\texttt{date}\,\rrbracket \parallel \texttt{C}\llbracket\, \texttt{A}?\,\texttt{card}\,;\, \bullet\rangle\!\rangle\circ\texttt{C}\,\rrbracket$$
$$\Downarrow$$
$$\texttt{A}\llbracket\, \texttt{C}?\,\texttt{date}\,\rrbracket \parallel \overset{*}{\texttt{C}}\llbracket\, \texttt{B}\bullet\rangle\!\rangle\circ;\, \texttt{A}!\,\texttt{date}\,\rrbracket \parallel \texttt{C}\llbracket\, \bullet\rangle\!\rangle\circ\texttt{C}\,\rrbracket$$
$$\Downarrow$$
$$\texttt{A}\llbracket\, \texttt{C}?\,\texttt{date}\,\rrbracket \parallel \texttt{C}\llbracket\, \texttt{A}!\,\texttt{date}\,\rrbracket \parallel \texttt{B}\llbracket\, \mathbf{0}\,\rrbracket$$

$$\mathbb{N} \quad ::= \quad \mathsf{p}\llbracket\, P\,\rrbracket \quad | \quad \overset{*}{\mathsf{p}}\llbracket\, P\,\rrbracket \quad | \quad \mathbb{N} \parallel \mathbb{N}$$

Operational Semantics

$$\Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; P_i \xrightarrow{\mathsf{p}_j?\Lambda_j} P_j \quad j \in I \qquad [\textsc{ExtCh}]$$

Operational Semantics

$$\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \xrightarrow{\mathsf{p}_j?\Lambda_j} P_j \quad j \in I \qquad [\textsc{ExtCh}]$$

$$\bigoplus_{i\in I}\mathsf{p}_i!\Lambda_i; P_i \xrightarrow{\mathsf{p}_j!\Lambda_j} P_j \quad j \in I \qquad [\textsc{IntCh}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \text{ [COM]}$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \; [\textsc{Com}]$$

$$\mathsf{p}[\![\, \circ\langle\!\langle \bullet \mathsf{q}; P \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p} \circ\langle\!\langle \bullet; Q \,]\!] \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \overset{*}{\mathsf{p}}[\![\, P \,]\!] \parallel \mathsf{p}[\![\, Q \,]\!] \quad [\textsc{BDel}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \ [\textsc{Com}]$$

$$\mathsf{p}[\![\, \circ\langle\!\langle\bullet \mathsf{q}; P \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p}\circ\langle\!\langle\bullet; Q \,]\!] \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \overset{*}{\mathsf{p}}[\![\, P \,]\!] \parallel \mathsf{p}[\![\, Q \,]\!] \quad [\textsc{BDel}]$$

$$\overset{*}{\mathsf{p}}[\![\, \mathsf{q}\bullet\rangle\!\rangle\circ; P \,]\!] \parallel \mathsf{p}[\![\, \bullet\rangle\!\rangle\circ\mathsf{p}; Q \,]\!] \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \quad [\textsc{EDel}]$$

Operational Semantics

$$\frac{P \xrightarrow{\mathsf{q}!\Lambda} P' \quad Q \xrightarrow{\mathsf{p}?\Lambda} Q'}{\mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathsf{p}[\![\, P' \,]\!] \parallel \mathsf{q}[\![\, Q' \,]\!]} \; [\textsc{Com}]$$

$$\mathsf{p}[\![\, \circ\langle\!\langle \bullet \mathsf{q}; P \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p} \circ\langle\!\langle \bullet; Q \,]\!] \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \overset{*}{\mathsf{p}}[\![\, P \,]\!] \parallel \mathsf{p}[\![\, Q \,]\!] \quad [\textsc{BDel}]$$

$$\overset{*}{\mathsf{p}}[\![\, \mathsf{q} \bullet\rangle\!\rangle\circ; P \,]\!] \parallel \mathsf{p}[\![\, \bullet\rangle\!\rangle\circ\mathsf{p}; Q \,]\!] \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathsf{p}[\![\, P \,]\!] \parallel \mathsf{q}[\![\, Q \,]\!] \quad [\textsc{EDel}]$$

$$\frac{\mathbb{N} \xrightarrow{\phi} \mathbb{N}'}{\mathbb{N} \parallel \mathbb{N}'' \xrightarrow{\phi} \mathbb{N}' \parallel \mathbb{N}''} \; [\textsc{Ct}]$$

ϕ ranges over $\mathsf{p}\Lambda\mathsf{q}$, $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$, $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Partial Order on Processes

a process offering more inputs and less outputs is better

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[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

Partial Order on Processes

[SUB-IN]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[SUB-OUT]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

connecting communications are better than $\mathbf{0}$

Partial Order on Processes

[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

connecting communications are better than **0**

[Sub-In-Skip]

$$\frac{}{\Sigma_{i \in I} \mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i \leq \mathbf{0}}$$

Partial Order on Processes

[Sub-In]

$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i?\Lambda_i; P_i \leq \Sigma_{i \in I} \mathsf{p}_i?\Lambda_i; Q_i}$$

[Sub-Out]

$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i!\Lambda_i; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i!\Lambda_i; Q_i}$$

[Sub-In-Skip]

$$\frac{}{\Sigma_{i \in I} \mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i \leq \mathbf{0}}$$

δ *ranges over* p∘⟪• ∘⟪•q q•⟫∘ •⟫∘p

Partial Order on Processes

[Sub-In]
$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i ? \Lambda_i ; P_i \leq \Sigma_{i \in I} \mathsf{p}_i ? \Lambda_i ; Q_i}$$

[Sub-Out]
$$\frac{\forall i \in I : P_i \leq Q_i}{\bigoplus_{i \in I} \mathsf{p}_i ! \Lambda_i ; P_i \leq \bigoplus_{i \in I \cup J} \mathsf{p}_i ! \Lambda_i ; Q_i}$$

[Sub-In-Skip]
$$\frac{}{\Sigma_{i \in I} \mathsf{p}_i ? \overset{\lambda_i}{\leftrightarrow} ; P_i \leq \mathbf{0}}$$

δ *ranges over* $\mathsf{p}\circ\langle\!\langle\bullet \quad \circ\langle\!\langle\bullet\mathsf{q} \quad \mathsf{q}\bullet\rangle\!\rangle\circ \quad \bullet\rangle\!\rangle\circ\mathsf{p}$

[Sub-Del]
$$\frac{P \leq Q}{\delta; P \leq \delta; Q}$$

Partial Order on Processes

[Sub-In]
$$\frac{\forall i \in I : P_i \leq Q_i}{\Sigma_{i \in I \cup J} \mathsf{p}_i ? \Lambda_i ; P_i \leq \Sigma_{i \in I} \mathsf{p}_i ? \Lambda_i ; Q_i}$$

[Sub-Out]
$$\frac{\forall i \in I : P_i \leq Q_i}{\oplus_{i \in I} \mathsf{p}_i ! \Lambda_i ; P_i \leq \oplus_{i \in I \cup J} \mathsf{p}_i ! \Lambda_i ; Q_i}$$

[Sub-In-Skip]
$$\frac{}{\Sigma_{i \in I} \mathsf{p}_i ? \overset{\lambda_i}{\leftrightarrow} ; P_i \leq \mathbf{0}}$$

[Sub-Del]
$$\frac{P \leq Q}{\delta ; P \leq \delta ; Q}$$

[Sub-**0**]
$$\mathbf{0} \leq \mathbf{0}$$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad \color{red}{\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B}};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad \color{blue}{\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\qquad \boxplus\\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \quad)
\end{array}
$$

Global types

$$
\begin{aligned}
&\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
&\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
&\quad (\ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
&\quad\ \ \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
&\quad\ \ \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
&\quad\ \ \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
&\quad\ \ \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
&\quad\ \ \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
&\quad\ \ \boxplus\\
&\quad\ \ \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End}\ )
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{G} \quad ::= \quad & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i \mathsf{q}_i; \mathsf{G}_i\\
& \mid\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G} \ \mid\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}\\
& \mid\ \mu\mathbf{t}.\mathsf{G} \ \mid\ \mathbf{t} \ \mid\ \mathsf{End}
\end{aligned}
$$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\quad\quad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\quad\quad \mathtt{C} \circ\langle\!\langle \bullet \mathtt{B};\\
\quad\quad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\quad\quad \mathtt{B} \bullet\rangle\!\rangle \circ \mathtt{C};\\
\quad\quad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\quad\quad \boxplus\\
\quad\quad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End}\ \ )
\end{array}
$$

$$
\begin{array}{rcl}
\mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\\
 & | & \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G} \ \ | \ \ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}\\
 & | & \mu\mathbf{t}.\mathsf{G} \ \ | \ \ \mathbf{t} \ \ | \ \ \mathsf{End}
\end{array}
$$

- no ambiguity of choices between all simple or all connecting communications

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C}; \\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A}; \\
\qquad (\;\; \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C}; \\
\qquad\quad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B}; \\
\qquad\quad \mathtt{C} \circ\langle\langle \bullet \mathtt{B}; \\
\qquad\quad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}; \\
\qquad\quad \mathtt{B} \bullet\rangle\rangle \circ \mathtt{C}; \\
\qquad\quad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End} \\
\qquad\quad \boxplus \\
\qquad\quad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \;\;)
\end{array}
$$

$$
\begin{array}{lll}
\mathsf{G} & ::= & \boxplus_{i \in I}\, \mathsf{p} \Lambda_i \mathsf{q}_i; \mathsf{G}_i \\
 & & \mid\; \mathsf{p} \circ\langle\langle \bullet \mathsf{q}; \mathsf{G} \;\mid\; \mathsf{q} \bullet\rangle\rangle \circ \mathsf{p}; \mathsf{G} \\
 & & \mid\; \mu \mathbf{t}.\mathsf{G} \;\mid\; \mathbf{t} \;\mid\; \mathsf{End}
\end{array}
$$

- no ambiguity of choices between all simple or all connecting communications
- each occurrence of $\mathsf{p} \circ\langle\langle \bullet \mathsf{q}$ is followed by an occurrence of $\mathsf{q} \bullet\rangle\rangle \circ \mathsf{p}$

Global types

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\ \mathsf{End}\\
\qquad \boxplus\\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\ \mathsf{End}\ \ )
\end{array}
$$

$$
\begin{array}{lll}
\mathsf{G} & ::= & \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i;\, \mathsf{G}_i \\
 & | & \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q};\, \mathsf{G} \ \ | \ \ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p};\, \mathsf{G} \\
 & | & \mu\mathbf{t}.\mathsf{G} \ \ | \ \ \mathbf{t} \ \ | \ \ \mathsf{End}
\end{array}
$$

- no ambiguity of choices between all simple or all connecting communications
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no atomic interaction involving q occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Global types

$$\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};$$
$$(\quad \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};$$
$$\mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};$$
$$\mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};$$
$$\mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};$$
$$\mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};$$
$$\mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A};\, \mathsf{End}$$
$$\boxplus$$
$$\mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C};\, \mathsf{End} \quad)$$

$$\mathsf{G} \ ::= \ \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i;\, \mathsf{G}_i \ \mid\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q};\, \mathsf{G} \ \mid\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p};\, \mathsf{G} \ \mid\ \mu\mathbf{t}.\mathsf{G} \ \mid\ \mathbf{t} \ \mid\ \mathsf{End}$$

- no ambiguity of choices between all simple or all connecting communications
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no atomic interaction involving q occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no choice occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Global types

$$\mathsf{A} \xrightarrow{\texttt{title}} \mathsf{C};$$
$$\mathsf{C} \xrightarrow{\texttt{price}} \mathsf{A};$$
$$(\ \mathsf{A} \xrightarrow{\texttt{ok}} \mathsf{C};$$
$$\mathsf{C} \xrightarrow{\overset{\texttt{price}}{\leftrightarrow}} \mathsf{B};$$
$$\mathsf{C}\circ\langle\!\langle\bullet\mathsf{B};$$
$$\mathsf{A} \xrightarrow{\texttt{card}} \mathsf{C};$$
$$\mathsf{B}\bullet\rangle\!\rangle\circ\mathsf{C};$$
$$\mathsf{C} \xrightarrow{\texttt{date}} \mathsf{A};\ \mathsf{End}$$
$$\boxplus$$
$$\mathsf{A} \xrightarrow{\texttt{ko}} \mathsf{C};\ \mathsf{End}\ )$$

$$\mathsf{G} ::= \boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i;\, \mathsf{G}_i \mid \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q};\mathsf{G} \mid \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p};\mathsf{G} \mid \mu\mathbf{t}.\mathsf{G} \mid \mathbf{t} \mid \mathsf{End}$$

- no ambiguity of choices between all simple or all connecting communications
- each occurrence of $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ is followed by an occurrence of $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no atomic interaction involving q occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no choice occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$
- no delegation involving p occurs between $\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}$ and $\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}$

Projection: Example

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C};\\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A};\\
\quad (\ \ \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B};\\
\qquad \mathtt{C}\circ\langle\!\langle\bullet\mathtt{B};\\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C};\\
\qquad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C};\\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End}\\
\qquad \boxplus\\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End}\ \ )
\end{array}
$$

Projection: Example

$$\begin{array}{ll}
\texttt{A?title}; & \texttt{A} \xrightarrow{\texttt{title}} \texttt{C}; \\
\texttt{A!price}; & \texttt{C} \xrightarrow{\texttt{price}} \texttt{A}; \\
\quad (\ \texttt{A?ok}; & \quad (\ \texttt{A} \xrightarrow{\texttt{ok}} \texttt{C}; \\
\qquad \texttt{B!}\overset{\texttt{price}}{\leftrightarrow}; & \qquad \texttt{C} \xrightarrow{\overset{\texttt{price}}{\leftrightarrow}} \texttt{B}; \\
\qquad \circ\langle\!\langle \bullet \texttt{B}; & \qquad \texttt{C}\circ\langle\!\langle \bullet \texttt{B}; \\
 & \qquad \texttt{A} \xrightarrow{\texttt{card}} \texttt{C}; \\
\qquad \texttt{B}\bullet\rangle\!\rangle\circ; & \qquad \texttt{B}\bullet\rangle\!\rangle\circ\texttt{C}; \\
\qquad \texttt{A!date}; & \qquad \texttt{C} \xrightarrow{\texttt{date}} \texttt{A}; \mathsf{End} \\
\qquad + & \qquad \boxplus \\
\qquad \texttt{A?ko}\ ) & \qquad \texttt{A} \xrightarrow{\texttt{ko}} \texttt{C}; \mathsf{End}\ )
\end{array}$$

Projection: Example

$$
\begin{array}{l}
\mathtt{A}\,?\,\mathtt{title}\,; \\
\mathtt{A}\,!\,\mathtt{price}\,; \\
\quad (\;\; \mathtt{A}\,?\,\mathtt{ok}\,; \\
\qquad \mathtt{B}\,!\overset{\mathtt{price}}{\leftrightarrow}\,; \\
\qquad \circ\langle\!\langle \bullet \mathtt{B}; \\
\\
\qquad \mathtt{B}\bullet\rangle\!\rangle\circ; \\
\qquad \mathtt{A}\,!\,\mathtt{date}\,; \\
\qquad + \\
\qquad \mathtt{A}\,?\,\mathtt{ko} \quad)
\end{array}
$$

$$
\begin{array}{l}
\mathtt{A} \xrightarrow{\mathtt{title}} \mathtt{C}; \\
\mathtt{C} \xrightarrow{\mathtt{price}} \mathtt{A}; \\
\quad (\;\; \mathtt{A} \xrightarrow{\mathtt{ok}} \mathtt{C}; \\
\qquad \mathtt{C} \xrightarrow{\overset{\mathtt{price}}{\leftrightarrow}} \mathtt{B}; \\
\qquad \mathtt{C}\circ\langle\!\langle \bullet \mathtt{B}; \\
\qquad \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}; \\
\qquad \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}; \\
\qquad \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End} \\
\qquad \boxplus \\
\qquad \mathtt{A} \xrightarrow{\mathtt{ko}} \mathtt{C}; \mathsf{End} \quad)
\end{array}
$$

$$
\begin{array}{l}
\mathtt{C}\circ\langle\!\langle \bullet; \\
\mathtt{A}\,?\,\mathtt{card}\,; \\
\bullet\rangle\!\rangle\circ\mathtt{C}
\end{array}
$$

Direct Projection

Meet

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \; + \; \mathsf{p}?\Lambda; P$$

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i)\sqcap \mathsf{p}?\Lambda;P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i \;+\; \mathsf{p}?\Lambda;P$$

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i)\sqcap \mathsf{p}?\Lambda;P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i;P_i$$

$$\text{if } \mathsf{p}=\mathsf{p}_j \text{ and } \Lambda=\Lambda_j \text{ and } P=P_j \text{ for some } j\in I$$

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i)\sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \ + \ \mathsf{p}?\Lambda; P$$

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i)\sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i$$

$$\text{if } \mathsf{p} = \mathsf{p}_j \text{ and } \Lambda = \Lambda_j \text{ and } P = P_j \text{ for some } j \in I$$

$$(\Sigma_{i\in I}\mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i)\sqcap \mathbf{0} = \Sigma_{i\in I}\mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i$$

Direct Projection

Meet

$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i \;+\; \mathsf{p}?\Lambda; P$$
$$(\Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i) \sqcap \mathsf{p}?\Lambda; P = \Sigma_{i\in I}\mathsf{p}_i?\Lambda_i; P_i$$
$$\text{if } \mathsf{p} = \mathsf{p}_j \text{ and } \Lambda = \Lambda_j \text{ and } P = P_j \text{ for some } j \in I$$
$$(\Sigma_{i\in I}\mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i) \sqcap \mathbf{0} = \Sigma_{i\in I}\mathsf{p}_i?\overset{\lambda_i}{\leftrightarrow}; P_i$$
$$\mathbf{0} \sqcap \mathbf{0} = \mathbf{0}$$

Direct Projection

$$(\, p \Lambda q; G \,) \upharpoonright r = \begin{cases} q!\Lambda; G \upharpoonright p & \text{if } r = p \\ p?\Lambda; G \upharpoonright q & \text{if } r = q \\ G \upharpoonright r & \text{if } r \notin \{p, q\} \end{cases}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q};\mathsf{G}\ )\upharpoonright\mathsf{r} = \begin{cases} \mathsf{q}!\Lambda;\mathsf{G}\upharpoonright\mathsf{p} & \text{if } \mathsf{r}=\mathsf{p} \\ \mathsf{p}?\Lambda;\mathsf{G}\upharpoonright\mathsf{q} & \text{if } \mathsf{r}=\mathsf{q} \\ \mathsf{G}\upharpoonright\mathsf{r} & \text{if } \mathsf{r}\notin\{\mathsf{p},\mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i\in I}\,\mathsf{p}\Lambda_i\mathsf{q}_i;\mathsf{G}_i\ )\upharpoonright\mathsf{r} = \begin{cases} \bigoplus_{i\in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i;\mathsf{G}_i\ )\upharpoonright\mathsf{r} & \text{if } \mathsf{r}=\mathsf{p} \\ \prod_{i\in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i;\mathsf{G}_i\ )\upharpoonright\mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I|>1$$

Direct Projection

$$(\ \mathsf{p} \Lambda \mathsf{q}; \mathsf{G}\ ) \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i \in I} \mathsf{p} \Lambda_i \mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} = \begin{cases} \bigoplus_{i \in I}(\ \mathsf{p} \Lambda_i \mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i \in I}(\ \mathsf{p} \Lambda_i \mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu \mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu \mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}\ ) \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} = \begin{cases} \bigoplus_{i \in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i \in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \upharpoonright \mathsf{p} = \mathbf{t}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}\ ) \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \upharpoonright \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} = \begin{cases} \bigoplus_{i \in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i \in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \upharpoonright \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \upharpoonright \mathsf{p} = \begin{cases} \mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \upharpoonright \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \upharpoonright \mathsf{p} = \mathbf{t} \qquad \mathsf{End} \upharpoonright \mathsf{p} = \mathbf{0}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}\ ) \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \restriction \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \restriction \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \restriction \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i\in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \restriction \mathsf{r} = \begin{cases} \bigoplus_{i\in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \restriction \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i\in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \restriction \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \restriction \mathsf{p} = \begin{cases} \mathsf{G} \restriction \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \restriction \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \restriction \mathsf{p} = \mathbf{t} \qquad \mathsf{End} \restriction \mathsf{p} = \mathbf{0}$$

$$(\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}) \restriction \mathsf{r} = \begin{cases} \circ\langle\!\langle\bullet\ \mathsf{q}; \mathsf{G} \restriction_1(\mathsf{p}, \mathsf{q}) & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}\circ\langle\!\langle\bullet; \mathsf{G} \restriction_2(\mathsf{p}, \mathsf{q}) & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \restriction \mathsf{r} & \text{otherwise} \end{cases}$$

Direct Projection

$$(\ \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}\ ) \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\Lambda; \mathsf{G} \restriction \mathsf{p} & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}?\Lambda; \mathsf{G} \restriction \mathsf{q} & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \restriction \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \end{cases}$$

$$(\ \boxplus_{i\in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \restriction \mathsf{r} = \begin{cases} \bigoplus_{i\in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \restriction \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \prod_{i\in I}(\ \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i\ ) \restriction \mathsf{r} & \text{otherwise} \end{cases} \quad \text{where } |I| > 1$$

$$(\mu\mathbf{t}.\mathsf{G}) \restriction \mathsf{p} = \begin{cases} \mathsf{G} \restriction \mathsf{p} & \text{if } \mathbf{t} \text{ does not occur in } \mathsf{G} \\ \mu\mathbf{t}.\mathsf{G} \restriction \mathsf{p} & \text{if } \mathsf{p} \in \mathsf{part}(\mathsf{G}) \\ \mathbf{0} & \text{otherwise} \end{cases} \qquad \mathbf{t} \restriction \mathsf{p} = \mathbf{t} \qquad \mathsf{End} \restriction \mathsf{p} = \mathbf{0}$$

$$(\mathsf{p}{\circ}\langle\!\langle{\bullet}\mathsf{q}; \mathsf{G}) \restriction \mathsf{r} = \begin{cases} {\circ}\langle\!\langle{\bullet}\, \mathsf{q}; \mathsf{G} \restriction_1(\mathsf{p}, \mathsf{q}) & \text{if } \mathsf{r} = \mathsf{p} \\ \mathsf{p}{\circ}\langle\!\langle{\bullet}; \mathsf{G} \restriction_2(\mathsf{p}, \mathsf{q}) & \text{if } \mathsf{r} = \mathsf{q} \\ \mathsf{G} \restriction \mathsf{r} & \text{otherwise} \end{cases}$$

$$(\mathsf{q}{\bullet}\rangle\!\rangle{\circ}\mathsf{p}; \mathsf{G}) \restriction \mathsf{r} = \mathsf{G} \restriction \mathsf{r} \qquad \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}$$

Delegation Projection

$$(\; r\Lambda s; G \;) \restriction_2 (p, q) = \begin{cases} s!\Lambda; G \restriction_2 (p, q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2 (p, q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2 (p, q) & \text{if } \{r, s\} \cap \{p, q\} = \emptyset \end{cases}$$

Delegation Projection

$$(\ r\Lambda s; G\ ) \upharpoonright_2 (p, q) = \begin{cases} s!\Lambda; G \upharpoonright_2 (p, q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \upharpoonright_2 (p, q) & \text{if } s = p \text{ and } r \neq q \\ G \upharpoonright_2 (p, q) & \text{if } \{r, s\} \cap \{p, q\} = \emptyset \end{cases}$$

$$(\ r\Lambda s; G\ ) \upharpoonright_1 (p, q) = G \upharpoonright_1 (p, q) \quad \text{if } r \neq q \text{ and } s \neq q$$

Delegation Projection

$$(\ r\Lambda s; G \ ) \restriction_2(p,q) = \begin{cases} s!\Lambda; G \restriction_2(p,q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2(p,q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2(p,q) & \text{if } \{r,s\} \cap \{p,q\} = \emptyset \end{cases}$$

$$(\ r\Lambda s; G \ ) \restriction_1(p,q) = G \restriction_1(p,q) \quad \text{if } r \neq q \text{ and } s \neq q$$

$$(q\bullet\rangle\!\rangle\circ p; G) \restriction_1(p,q) = q\bullet\rangle\!\rangle\circ; G \restriction p \qquad (q\bullet\rangle\!\rangle\circ p; G) \restriction_2(p,q) = \bullet\rangle\!\rangle\circ p; G \restriction q$$

Delegation Projection

$$(\; r \Lambda s; G \;) \restriction_2 (p, q) = \begin{cases} s!\Lambda; G \restriction_2 (p, q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2 (p, q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2 (p, q) & \text{if } \{r, s\} \cap \{p, q\} = \emptyset \end{cases}$$

$$(\; r \Lambda s; G \;) \restriction_1 (p, q) = G \restriction_1 (p, q) \quad \text{if } r \neq q \text{ and } s \neq q$$

$$(q \bullet \rangle\!\rangle \circ p; G) \restriction_1 (p, q) = q \bullet \rangle\!\rangle \circ; G \restriction p \qquad (q \bullet \rangle\!\rangle \circ p; G) \restriction_2 (p, q) = \bullet \rangle\!\rangle \circ p; G \restriction q$$

$$(r \circ \langle\!\langle \bullet s; G) \restriction_1 (p, q) = (r \bullet \rangle\!\rangle \circ s; G) \restriction_1 (p, q) = G \restriction_1 (p, q) \quad \text{if } \{r, s\} \cap \{p, q\} = \emptyset$$

Delegation Projection

$$(\; r\Lambda s; G \;) \restriction_2 (p,q) \; = \begin{cases} s!\Lambda; G \restriction_2 (p,q) & \text{if } r = p \text{ and } s \neq q \\ r?\Lambda; G \restriction_2 (p,q) & \text{if } s = p \text{ and } r \neq q \\ G \restriction_2 (p,q) & \text{if } \{r,s\} \cap \{p,q\} = \emptyset \end{cases}$$

$$(\; r\Lambda s; G \;) \restriction_1 (p,q) \; = G \restriction_1 (p,q) \quad \text{if } r \neq q \text{ and } s \neq q$$

$$(q\bullet\rangle\!\rangle\circ p; G) \restriction_1 (p,q) \; = q\bullet\rangle\!\rangle\circ; G \restriction p \qquad (q\bullet\rangle\!\rangle\circ p; G) \restriction_2 (p,q) \; = \; \bullet\rangle\!\rangle\circ p; G \restriction q$$

$$(r\circ\langle\!\langle\bullet s; G) \restriction_1 (p,q) \; = (r\bullet\rangle\!\rangle\circ s; G) \restriction_1 (p,q) \; = G \restriction_1 (p,q) \quad \text{if } \; \{r,s\} \cap \{p,q\} = \emptyset$$

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Typing Rule

$$\frac{\begin{array}{ll} \mathsf{q}_i \bullet\rangle\!\rangle \circ; P_i \leq \mathsf{G}\restriction_1(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ Q_i \leq \mathsf{G}\restriction_2(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ R_j \leq \mathsf{G}\restriction \mathsf{r}_j & (j \in J) \\ \multicolumn{2}{l}{\mathsf{part}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\} \cup \{\mathsf{q}_i \mid i \in I\} \cup \{\mathsf{r}_j \mid j \in J\} \text{ all participants distinct}} \end{array}}{\vdash \Pi_{i \in I} \overset{*}{\mathsf{p}}_i[\![\, \mathsf{q}_i \bullet\rangle\!\rangle \circ; P_i \,]\!] \parallel \Pi_{i \in I}\, \mathsf{p}_i[\![\, Q_i \,]\!] \parallel \Pi_{j \in J}\, \mathsf{r}_j[\![\, R_j \,]\!] : \mathsf{G}}$$

Typing Rule

$$\dfrac{\begin{array}{ll} \mathsf{q}_i \bullet\!\rangle\!\rangle \circ; P_i \leq \mathsf{G} \restriction_1 (\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ Q_i \leq \mathsf{G} \restriction_2 (\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ R_j \leq \mathsf{G} \restriction \mathsf{r}_j & (j \in J) \\ \multicolumn{2}{l}{\mathsf{part}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\} \cup \{\mathsf{q}_i \mid i \in I\} \cup \{\mathsf{r}_j \mid j \in J\} \text{ all participants distinct}} \end{array}}{\vdash \Pi_{i \in I} \overset{*}{\mathsf{p}_i} [\![\mathsf{q}_i \bullet\!\rangle\!\rangle \circ; P_i]\!] \parallel \Pi_{i \in I} \mathsf{p}_i [\![Q_i]\!] \parallel \Pi_{j \in J} \mathsf{r}_j [\![R_j]\!] : \mathsf{G}}$$

$$\mathtt{A}[\![\mathtt{C!\,card\,;C?\,date}]\!] \parallel \overset{*}{\mathtt{C}} [\![\mathtt{B} \bullet\!\rangle\!\rangle \circ; \mathtt{A!\,date}]\!] \parallel \mathtt{C}[\![\mathtt{A?\,card\,;} \bullet\!\rangle\!\rangle \circ \mathtt{C}]\!]$$

Typing Rule

$$\frac{\begin{array}{ll} \mathsf{q}_i \bullet\rangle\!\rangle\circ; P_i \le \mathsf{G}\restriction_1(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ Q_i \le \mathsf{G}\restriction_2(\mathsf{p}_i, \mathsf{q}_i) & (i \in I) \\ R_j \le \mathsf{G}\restriction \mathsf{r}_j & (j \in J) \\ \multicolumn{2}{l}{\mathsf{part}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\} \cup \{\mathsf{q}_i \mid i \in I\} \cup \{\mathsf{r}_j \mid j \in J\} \text{ all participants distinct}} \end{array}}{\vdash \Pi_{i\in I} \overset{*}{\mathsf{p}}_i[\![\, \mathsf{q}_i \bullet\rangle\!\rangle\circ; P_i \,]\!] \parallel \Pi_{i\in I}\, \mathsf{p}_i[\![\, Q_i \,]\!] \parallel \Pi_{j\in J}\, \mathsf{r}_j[\![\, R_j \,]\!] : \mathsf{G}}$$

$$\mathtt{A}[\![\, \mathtt{C}!\,\mathtt{card}\,;\mathtt{C}?\,\mathtt{date} \,]\!] \parallel \overset{*}{\mathtt{C}}[\![\, \mathtt{B}\bullet\rangle\!\rangle\circ; \mathtt{A}!\,\mathtt{date} \,]\!] \parallel \mathtt{C}[\![\, \mathtt{A}?\,\mathtt{card}\,; \bullet\rangle\!\rangle\circ\mathtt{C} \,]\!]$$

$$\begin{array}{l} \mathtt{A} \xrightarrow{\mathtt{card}} \mathtt{C}; \\ \mathtt{B}\bullet\rangle\!\rangle\circ\mathtt{C}; \\ \mathtt{C} \xrightarrow{\mathtt{date}} \mathtt{A}; \mathsf{End} \end{array}$$

Subject Reduction

If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\phi} \mathbb{N}'$, then $\vdash \mathbb{N}' : \mathsf{G}'$ for some G'.

Session Fidelity

- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; (\boxplus_{i\in I}\, \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \boxplus \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}')$, where ϕ_j for $1 \leq j \leq n$ is an atomic interaction not involving p and q.

Session Fidelity

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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.

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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.

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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \boxplus_{i \in I}\mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i$, then $\mathbb{N} = \mathsf{p}[\![\oplus_{i \in I'} \mathsf{q}_i!\Lambda_i; P_i]\!] \parallel \mathbb{N}_0$ with $I' \subseteq I$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}_i$ and $\vdash \mathbb{N}_i : \mathsf{G}_i$ for all $i \in I'$.

Session Fidelity

- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda\mathsf{q}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; (\boxplus_{i \in I} \mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i \boxplus \mathsf{p}\Lambda\mathsf{q}; \mathsf{G}')$, where ϕ_j for $1 \leq j \leq n$ is an atomic interaction not involving p and q.
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- If $\vdash \mathbb{N} : \mathsf{G}$ and $\mathbb{N} \xrightarrow{\mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$, then $\mathsf{G} = \phi_1; \ldots; \phi_n; \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}; \mathsf{G}'$, where ϕ_i for $1 \leq i \leq n$ is an atomic interaction not involving p and q.
- If $\vdash \mathbb{N} : \boxplus_{i \in I}\mathsf{p}\Lambda_i\mathsf{q}_i; \mathsf{G}_i$, then $\mathbb{N} = \mathsf{p}[\![\oplus_{i \in I'} \mathsf{q}_i!\Lambda_i; P_i]\!] \parallel \mathbb{N}_0$ with $I' \subseteq I$ and $\mathbb{N} \xrightarrow{\mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}_i$ and $\vdash \mathbb{N}_i : \mathsf{G}_i$ for all $i \in I'$.
- If $\vdash \mathbb{N} : \phi; \mathsf{G}$, then $\mathbb{N} \xrightarrow{\phi} \mathbb{N}'$ and $\vdash \mathbb{N}' : \mathsf{G}$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\oplus_{i\in I}\mathsf{q}_i!\Lambda_i; P_i]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\overrightarrow{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\overrightarrow{\phi}$ and for all $i \in I$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\, \oplus_{i \in I} \mathsf{q}_i!\Lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\vec{\phi}$ and for all $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\, \Sigma_{i \in I} \mathsf{q}_i?\lambda_i; P_i \,]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{q}_i\lambda_i\mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and for some $i \in I$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\oplus_{i \in I} \mathsf{q}_i ! \Lambda_i ; P_i]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\Lambda_i \mathsf{q}_i} \mathbb{N}'$ for some $\vec{\phi}$ and for all $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\Sigma_{i \in I} \mathsf{q}_i ? \lambda_i ; P_i]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{q}_i \lambda_i \mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and for some $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\circ\langle\!\langle \bullet \mathsf{q} ; P]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\circ\langle\!\langle \bullet \mathsf{q}\ \vec{\phi'}\ \mathsf{q} \bullet \rangle\!\rangle \circ \mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and $\vec{\phi'}$.

Strong Progress

- If $\mathbb{N} = \mathsf{p}[\![\oplus_{i \in I} \mathsf{q}_i!\Lambda_i; P_i]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\Lambda_i\mathsf{q}_i} \mathbb{N}'$ for some $\vec{\phi}$ and for all $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\Sigma_{i \in I} \mathsf{q}_i?\lambda_i; P_i]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{q}_i\lambda_i\mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and for some $i \in I$.
- If $\mathbb{N} = \mathsf{p}[\![\circ\langle\!\langle \bullet \mathsf{q}; P]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}\ \vec{\phi}'\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and $\vec{\phi}'$.
- If $\mathbb{N} = \mathsf{q}[\![\mathsf{p}\circ\langle\!\langle \bullet; Q]\!] \parallel \mathbb{N}_0$, then $\mathbb{N} \xrightarrow{\vec{\phi}\ \mathsf{p}\circ\langle\!\langle\bullet\mathsf{q}\ \vec{\phi}'\ \mathsf{q}\bullet\rangle\!\rangle\circ\mathsf{p}} \mathbb{N}'$ for some $\vec{\phi}$ and $\vec{\phi}'$.

Internal versus Channel Delegation

- pro

Internal versus Channel Delegation

- pro
  - internal delegation allows a better control of the whole conversation

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- pro
  - internal delegation allows a better control of the whole conversation
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- con

Internal versus Channel Delegation

- pro
  - internal delegation allows a better control of the whole conversation
  - internal delegation assures progress with a simple type system
- con
  - channel delegation can represent more protocols

Future Work

- global types allowing

Future Work

- global types allowing
  - nested delegation

Future Work

- global types allowing
  - nested delegation
  - deputies to make choices

Future Work

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  - . . .

Future Work

- global types allowing
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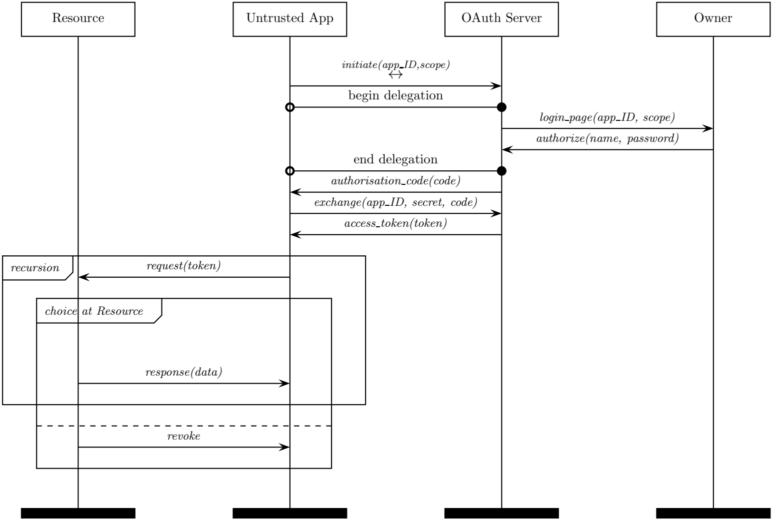
  - ...
- coherence of sets of session types

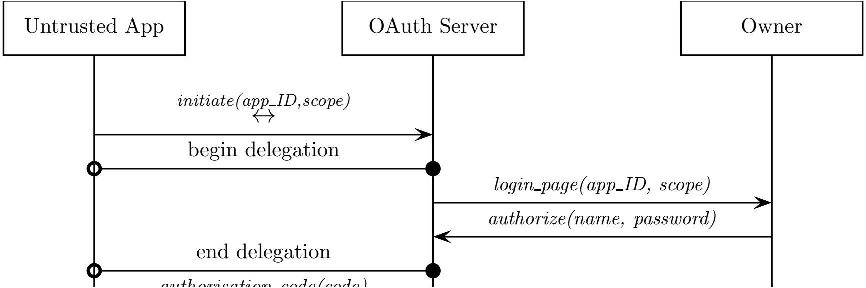
Future Work

- global types allowing
  - nested delegation
  - deputies to make choices
  - ...
- coherence of sets of session types
- integration with reversibility

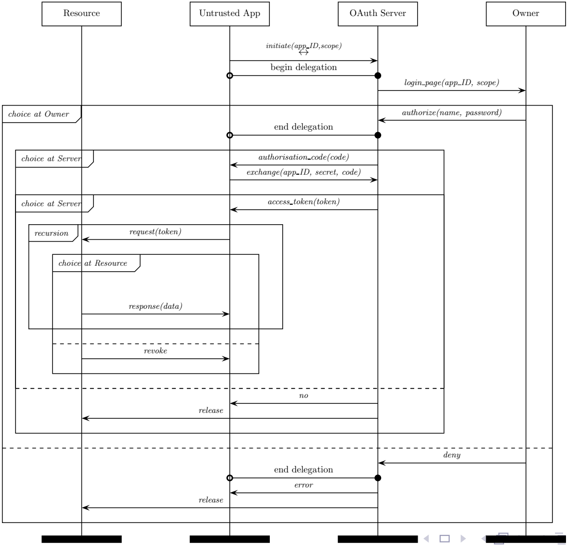
An application that delegates to an Oauth 2.0 server



An application that delegates to an Oauth 2.0 server



Allowing the deputy to make a choice is useful



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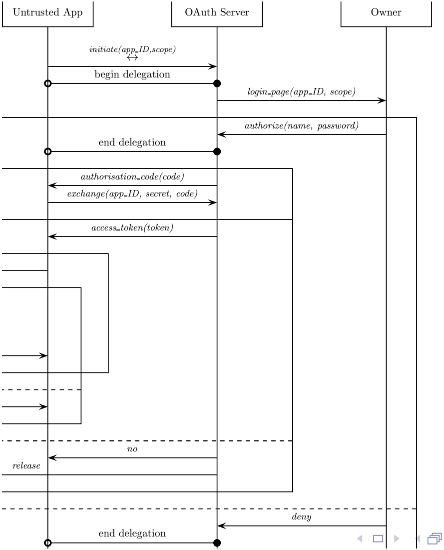

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Questions

Thank you


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Dank U
Merci
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Thank you
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