Effect handler oriented programming

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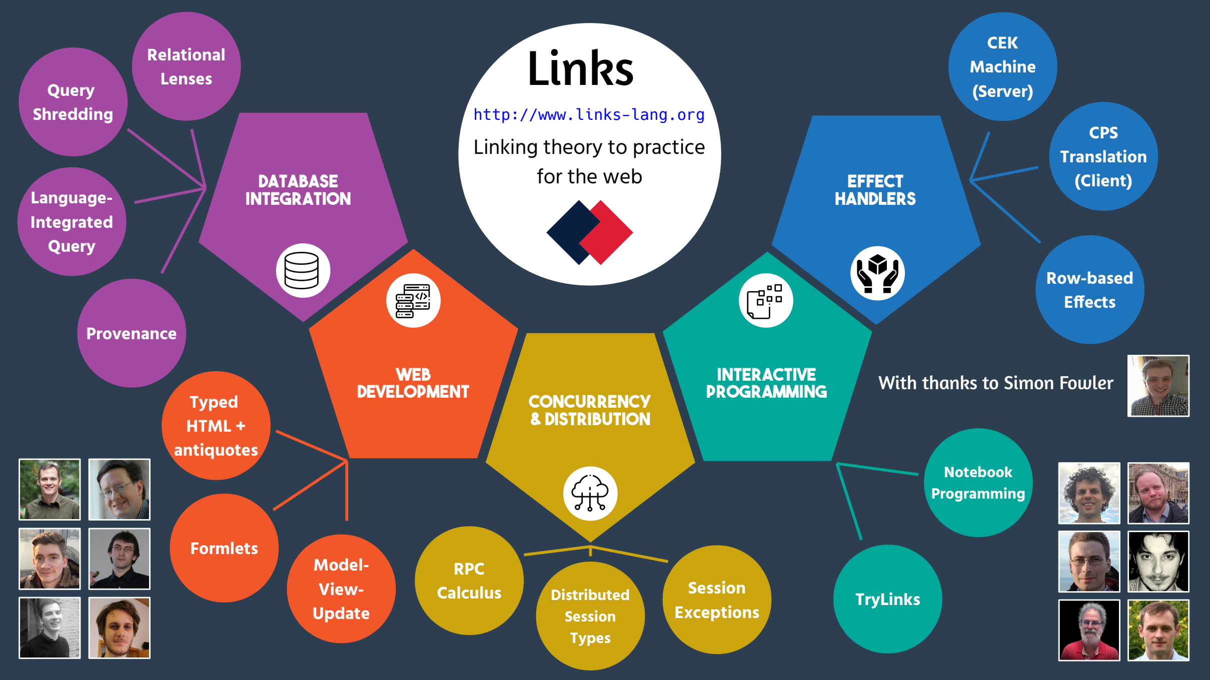
Part I

Prologue



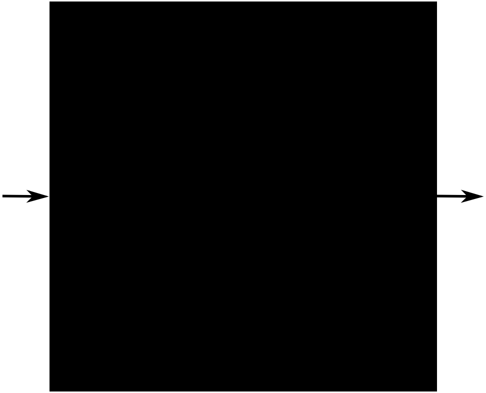
Links
http://www.links-lang.org
Linking theory to practice for the web
DATABASE INTEGRATION
Relational Lenses
Query Shredding
Language-Integrated Query
Provenance
WEB DEVELOPMENT
Typed HTML + antiquotes
Formlets
Model-View-Update
CONCURRENCY & DISTRIBUTION
RPC Calculus
Distributed Session Types
Session Exceptions
INTERACTIVE PROGRAMMING
Notebook Programming
TryLinks
EFFECT HANDLERS
CEK Machine (Server)
CPS Translation (Client)
Row-based Effects
With thanks to Simon Fowler

Part II

Effect handler oriented programming

Effects

Programs as black boxes (Church-Turing model)?



Effects

Programs must interact with their environment

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Effects are pervasive

- input/output
  user interaction
- concurrency
  web applications
- distribution
  cloud computing
- exceptions
  fault tolerance
- choice
  backtracking search

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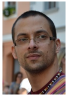
Typically ad hoc and hard-wired

Effect handlers

Deep theory

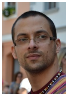
Gordon Plotkin



Matija Pretnar

Handlers of algebraic effects, ESOP 2009

Effect handlers

Deep theory  Gordon Plotkin  Matija Pretnar

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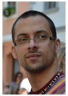
Composable and **user-defined** interpretation of effects in general

Effect handlers

Deep theory  Gordon Plotkin  Matija Pretnar

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Give programmer direct access to **environment**

(c.f. resumable exceptions, monads, delimited control)

Effect handlers

Deep theory


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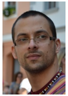
Matija Pretnar

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Growing industrial interest

	React	JavaScript UI library (used by $>$ 1 million websites)
Uber	Pyro	Probabilistic programming language (statistical inference)
GitHub	Semantic	Code analysis library ($>$ 4.5 million Python repositories)

Example 1: choice and failure

Effect signature

$$\{\mathsf{choose} : 1 \Rightarrow \mathsf{Bool}, \quad \mathsf{fail} : a.1 \Rightarrow a\}$$

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$$\{\text{choose} : 1 \Rightarrow \text{Bool}, \quad \text{fail} : a.1 \Rightarrow a\}$$

Drunk coin tossing

$$\text{toss}\,() = \textbf{if } \text{choose}\,() \textbf{ then } \text{Heads} \textbf{ else } \text{Tails}$$

$$\begin{aligned}\text{drunkToss}\,() = {} & \textbf{if } \text{choose}\,() \textbf{ then} \\ & \quad \textbf{if } \text{choose}\,() \textbf{ then } \text{Heads} \textbf{ else } \text{Tails} \\ & \textbf{else} \\ & \quad \text{fail}\,()\end{aligned}$$

$$\begin{aligned}\text{drunkTosses}\, n = {} & \textbf{if } n = 0 \textbf{ then } [] \\ & \textbf{else } \text{drunkToss}\,() :: \text{drunkTosses}\,(n-1)\end{aligned}$$

Example 1: choice and failure

Handlers

$$
\begin{aligned}
&\mathsf{maybeFail} = \qquad \text{— exception handler} \\
&\quad \mathbf{return}\, x \;\mapsto\; \mathsf{Just}\, x \\
&\quad \langle \mathsf{fail}\, () \rangle \;\mapsto\; \mathsf{Nothing}
\end{aligned}
$$

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$$\begin{aligned}
&\mathbf{handle}\ \ 42\ \ \mathbf{with}\ \mathsf{maybeFail} \Longrightarrow \mathsf{Just}\,42\\
&\mathbf{handle}\ \mathsf{fail}\,()\ \mathbf{with}\ \mathsf{maybeFail} \Longrightarrow \mathsf{Nothing}
\end{aligned}$$

Example 1: choice and failure

Handlers

$$\begin{aligned}
&\text{maybeFail} = &&\color{red}{\text{— exception handler}}\\
&\quad \textbf{return}\, x \;\mapsto\; \text{Just}\, x\\
&\quad \langle \color{blue}{\text{fail}}\, () \rangle \;\mapsto\; \text{Nothing}
\end{aligned}$$

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&\textbf{handle}\ \color{blue}{\text{fail}}\, ()\ \textbf{with}\ \text{maybeFail} \Longrightarrow \text{Nothing}
\end{aligned}$$

$$\begin{aligned}
&\text{trueChoice} = &&\color{red}{\text{— linear handler}}\\
&\quad \textbf{return}\, x \;\mapsto\; x\\
&\quad \langle \color{blue}{\text{choose}}\, () \to r \rangle \;\mapsto\; r\, \text{True}
\end{aligned}$$

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\end{aligned}$$

$$\textbf{handle}\ 42\ \textbf{with}\ \text{trueChoice} \Longrightarrow 42$$

$$\textbf{handle}\ \text{toss}\,()\ \textbf{with}\ \text{trueChoice} \Longrightarrow \text{Heads}$$

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$\text{allChoices} =$ — non-linear handler

$\quad \textbf{return}\, x \mapsto [x]$

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\end{array}$$

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\text{trueChoice} = & \text{— linear handler} \\
\quad \textbf{return}\, x \mapsto x & \textbf{handle}\ 42\ \textbf{with}\ \text{trueChoice} \Longrightarrow 42 \\
\quad \langle \text{choose}\,() \to r\rangle \mapsto r\ \text{True} & \textbf{handle}\ \text{toss}\,()\ \textbf{with}\ \text{trueChoice} \Longrightarrow \text{Heads}
\end{array}$$

$$\begin{array}{ll}
\text{allChoices} = & \text{— non-linear handler} \\
\quad \textbf{return}\, x \mapsto [x] & \textbf{handle}\ 42\ \textbf{with}\ \text{allChoices} \Longrightarrow [42] \\
\quad \langle \text{choose}\,() \to r\rangle \mapsto r\ \text{True} +\!\!+\ r\ \text{False} & \textbf{handle}\ \text{toss}\,()\ \textbf{with}\ \text{allChoices} \Longrightarrow [\text{Heads}, \text{Tails}]
\end{array}$$

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trueChoice = — linear handler

$$\begin{aligned} &\mathbf{return}\,x \mapsto x \\ &\langle \mathsf{choose}\,() \to r\rangle \mapsto r\,\mathsf{True} \end{aligned}$$

$$\begin{aligned} &\mathbf{handle}\ 42\ \mathbf{with}\ \mathsf{trueChoice} \Longrightarrow 42 \\ &\mathbf{handle}\ \mathsf{toss}\,()\ \mathbf{with}\ \mathsf{trueChoice} \Longrightarrow \mathsf{Heads} \end{aligned}$$

allChoices = — non-linear handler

$$\begin{aligned} &\mathbf{return}\,x \mapsto [x] \\ &\langle \mathsf{choose}\,() \to r\rangle \mapsto r\,\mathsf{True} + r\,\mathsf{False} \end{aligned}$$

$$\begin{aligned} &\mathbf{handle}\ 42\ \mathbf{with}\ \mathsf{allChoices} \Longrightarrow [42] \\ &\mathbf{handle}\ \mathsf{toss}\,()\ \mathbf{with}\ \mathsf{allChoices} \Longrightarrow [\mathsf{Heads}, \mathsf{Tails}] \end{aligned}$$

$$\begin{aligned} &\mathbf{handle}\ (\mathbf{handle}\ \mathsf{drunkTosses}\,2\ \mathbf{with}\ \mathsf{maybeFail})\ \mathbf{with}\ \mathsf{allChoices} \Longrightarrow \\ &\quad [\mathsf{Just}\,[\mathsf{Heads}, \mathsf{Heads}], \mathsf{Just}\,[\mathsf{Heads}, \mathsf{Tails}], \mathsf{Nothing}, \\ &\quad \mathsf{Just}\,[\mathsf{Tails}, \mathsf{Heads}], \mathsf{Just}\,[\mathsf{Tails}, \mathsf{Tails}], \mathsf{Nothing}, \mathsf{Nothing}] \end{aligned}$$

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\end{array}
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Small-step operational semantics for (deep) effect handlers

Reduction rules

$$
\begin{aligned}
\textbf{let } x = V \textbf{ in } N &\rightsquigarrow N[V/x] \\
\textbf{handle } V \textbf{ with } H &\rightsquigarrow N_{\mathrm{ret}}[V/x] \\
\textbf{handle } \mathcal{E}[\mathsf{op}\ V] \textbf{ with } H &\rightsquigarrow N_{\mathsf{op}}[V/p, (\lambda x.\textbf{handle } \mathcal{E}[x] \textbf{ with } H)/r], \quad \mathsf{op} \# \mathcal{E}
\end{aligned}
$$

$$
\begin{aligned}
\text{where } H = \textbf{return}\, x &\mapsto N_{\mathrm{ret}} \\
\langle \mathsf{op}_1\ p \rightarrow r \rangle &\mapsto N_{\mathsf{op}_1} \\
&\cdots \\
\langle \mathsf{op}_k\ p \rightarrow r \rangle &\mapsto N_{\mathsf{op}_k}
\end{aligned}
$$

Evaluation contexts

$$\mathcal{E} ::= [\,] \mid \textbf{let } x = \mathcal{E} \textbf{ in } N \mid \textbf{handle } \mathcal{E} \textbf{ with } H$$

Example 2: cooperative concurrency (static)

Effect signature

$$\{\mathsf{yield} : 1 \Rightarrow 1\}$$

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Two cooperative lightweight threads

$$\begin{aligned}
\text{tA}\,() &= \text{print}\,(\text{“A1 ”}); \text{yield}\,(); \text{print}\,(\text{“A2 ”}) \\
\text{tB}\,() &= \text{print}\,(\text{“B1 ”}); \text{yield}\,(); \text{print}\,(\text{“B2 ”})
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Handler — parameterised handler

$$\begin{array}{ll}
\text{coop}\,([]) = & \\
\quad \mathbf{return}\,() & \mapsto ()\\
\quad \langle \text{yield}\,() \to r' \rangle & \mapsto r'\,[]\,()
\end{array}
\qquad
\begin{array}{ll}
\text{coop}\,(r :: rs) = & \\
\quad \mathbf{return}\,() & \mapsto r\;rs\,()\\
\quad \langle \text{yield}\,() \to r' \rangle & \mapsto r\,(rs +\!\!+ [r'])\,()
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\text{coop}\,([]) = & \text{coop}\,(r :: rs) = \\
\quad \textbf{return}\,() \mapsto () & \quad \textbf{return}\,() \mapsto r\ rs\,() \\
\quad \langle \text{yield}\,() \to r' \rangle \mapsto r'\,[]\,() & \quad \langle \text{yield}\,() \to r' \rangle \mapsto r\,(rs +\!\!+ [r'])\,()
\end{array}$$

Helpers

$$\begin{aligned}
\text{coopWith}\ t\ rs\,() &= \textbf{handle}\ t\,()\ \textbf{with}\ \text{coop}\ rs \\
\text{cooperate}\ ts &= \text{coopWith}\ \text{id}\,(\text{map coopWith}\ ts)\,()
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\end{aligned}$$

$$\text{cooperate}\,[tA, tB] \Longrightarrow ()$$

A1 B1 A2 B2

Small-step operational semantics for parameterised effect handlers

Reduction rules

$$
\begin{aligned}
\textbf{let}\ x = V\ \textbf{in}\ N &\rightsquigarrow N[V/x] \\
\textbf{handle}\ V\ \textbf{with}\ H\ W &\rightsquigarrow N_{\mathrm{ret}}[V/x, W/h] \\
\textbf{handle}\ \mathcal{E}[\mathsf{op}\ V]\ \textbf{with}\ H\ W &\rightsquigarrow N_{\mathsf{op}}[V/p, W/h, (\lambda h\, x.\textbf{handle}\ \mathcal{E}[x]\ \textbf{with}\ H\ h)/r], \quad \mathsf{op} \# \mathcal{E}
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Evaluation contexts

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Small-step operational semantics for parameterised effect handlers

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Evaluation contexts

$$\mathcal{E} ::= [\,] \mid \textbf{let}\ x = \mathcal{E}\ \textbf{in}\ N \mid \textbf{handle}\ \mathcal{E}\ \textbf{with}\ H\ W$$

Exercise: express parameterised handlers as non-parameterised handlers

Example 3: cooperative concurrency (dynamic)

Effect signature — recursive effect signature

$$\mathsf{Co} = \{\mathsf{yield} : 1 \Rightarrow 1, \quad \mathsf{fork} : (1 \to [\mathsf{Co}]1) \Rightarrow 1\}$$

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$$\mathsf{Co} = \{\mathsf{yield} : 1 \Rightarrow 1, \quad \mathsf{fork} : (1 \to [\mathsf{Co}]1) \Rightarrow 1\}$$

A single cooperative program

$$\begin{aligned}\mathsf{main}\,() = {} & \mathsf{print}\ \text{“M1 ”}; \mathsf{fork}\,(\lambda().\mathsf{print}\ \text{“A1 ”}; \mathsf{yield}\,(); \mathsf{print}\ \text{“A2 ”}); \\ & \mathsf{print}\ \text{“M2 ”}; \mathsf{fork}\,(\lambda().\mathsf{print}\ \text{“B1 ”}; \mathsf{yield}\,(); \mathsf{print}\ \text{“B2 ”}); \mathsf{print}\ \text{“M3 ”}\end{aligned}$$

Example 3: cooperative concurrency (dynamic)

Effect signature — recursive effect signature

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$$\begin{aligned}\mathsf{main}\,() = {} & \mathsf{print}\ \text{“M1 ”}; \mathsf{fork}\,(\lambda().\mathsf{print}\ \text{“A1 ”}; \mathsf{yield}\,(); \mathsf{print}\ \text{“A2 ”}); \\ & \mathsf{print}\ \text{“M2 ”}; \mathsf{fork}\,(\lambda().\mathsf{print}\ \text{“B1 ”}; \mathsf{yield}\,(); \mathsf{print}\ \text{“B2 ”}); \mathsf{print}\ \text{“M3 ”}\end{aligned}$$

Parameterised handler and helpers

$$\begin{aligned}&\mathsf{coop}\,([\,]) = \\ &\quad \mathbf{return}\,() \mapsto () \\ &\quad \langle \mathsf{yield}\,() \to r' \rangle \mapsto r'\,[\,]\,() \\ &\quad \langle \mathsf{fork}\ t \to r' \rangle \mapsto \mathsf{coopWith}\ t\,[r']\,()\end{aligned}$$

$$\begin{aligned}&\mathsf{coop}\,(r :: rs) = \\ &\quad \mathbf{return}\,() \mapsto r\ rs\,() \\ &\quad \langle \mathsf{yield}\,() \to r' \rangle \mapsto r\,(rs \mathbin{+\!\!+} [r'])\,() \\ &\quad \langle \mathsf{fork}\ t \to r' \rangle \mapsto \mathsf{coopWith}\ t\,(r :: rs \mathbin{+\!\!+} [r'])\,()\end{aligned}$$

$$\begin{aligned}\mathsf{coopWith}\ t\ rs\,() &= \mathbf{handle}\ t\,()\ \mathbf{with}\ \mathsf{coop}\ rs \\ \mathsf{cooperate}\ ts &= \mathsf{coopWith}\ \mathsf{id}\,(\mathsf{map}\ \mathsf{coopWith}\ ts)\,()\end{aligned}$$

Example 3: cooperative concurrency (dynamic)

Effect signature — recursive effect signature

$$\mathsf{Co} = \{\mathsf{yield} : 1 \Rightarrow 1,\quad \mathsf{fork} : (1 \to [\mathsf{Co}]1) \Rightarrow 1\}$$

A single cooperative program

$$\begin{aligned}\mathsf{main}\,() = {} & \mathsf{print}\ \text{“M1 ”}; \mathsf{fork}\,(\lambda().\mathsf{print}\ \text{“A1 ”}; \mathsf{yield}\,(); \mathsf{print}\ \text{“A2 ”}); \\ & \mathsf{print}\ \text{“M2 ”}; \mathsf{fork}\,(\lambda().\mathsf{print}\ \text{“B1 ”}; \mathsf{yield}\,(); \mathsf{print}\ \text{“B2 ”}); \mathsf{print}\ \text{“M3 ”}\end{aligned}$$

Parameterised handler and helpers

$$\begin{aligned}&\mathsf{coop}\,([]) = \\ &\quad \mathbf{return}\,() \mapsto () \\ &\quad \langle \mathsf{yield}\,() \to r' \rangle \mapsto r'\,[]\,() \\ &\quad \langle \mathsf{fork}\ t \to r' \rangle \mapsto \mathsf{coopWith}\ t\,[r']\,()\end{aligned}$$

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$$\mathsf{cooperate}\,[\mathsf{main}] \Longrightarrow ()$$

M1 A1 M2 B1 A2 M3 B2

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$$\begin{aligned}&\mathsf{coop}\,([]) = \\ &\quad \mathbf{return}\,() \mapsto () \\ &\quad \langle \mathsf{yield}\,() \to r' \rangle \mapsto r'\,[]\,() \\ &\quad \langle \mathsf{fork}\; t \to r' \rangle \mapsto r'\,[\mathsf{coopWith}\; t]\,()\end{aligned}$$

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$$\mathsf{cooperate}\,[\mathsf{main}] \Longrightarrow ()$$

M1 M2 M3 A1 B1 A2 B2

Example 4: pipes

Effect signatures

$$\text{Sender} = \{\text{send} : \text{Nat} \Rightarrow 1\} \qquad \text{Receiver} = \{\text{receive} : 1 \Rightarrow \text{Nat}\}$$

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A producer and a consumer

$$\mathsf{nats}\, n = \mathsf{send}\, n; \mathsf{nats}\,(n+1) \qquad \mathsf{grabANat}\,() = \mathsf{receive}\,()$$

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Pipes and copipes as shallow handlers

$$\begin{aligned}\mathsf{pipe}\, p\, c = \mathbf{handle}^{\dagger}\ c\,()\ \mathbf{with}& \\ \mathbf{return}\, x \quad &\mapsto x \\ \langle \mathsf{receive}\,() \to r\rangle &\mapsto \mathsf{copipe}\, r\, p\end{aligned} \qquad \begin{aligned}\mathsf{copipe}\, c\, p = \mathbf{handle}^{\dagger}\ p\,()\ \mathbf{with}& \\ \mathbf{return}\, x \quad &\mapsto x \\ \langle \mathsf{send}\, n \to r\rangle &\mapsto \mathsf{pipe}\, r\,(\lambda().c\, n)\end{aligned}$$

Example 4: pipes

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Pipes and copipes as shallow handlers

$$\begin{array}{ll}
\mathsf{pipe}\, p\, c = \mathbf{handle}^{\dagger}\ c\,()\ \mathbf{with} & \mathsf{copipe}\, c\, p = \mathbf{handle}^{\dagger}\ p\,()\ \mathbf{with} \\
\quad \mathbf{return}\, x \mapsto x & \quad \mathbf{return}\, x \mapsto x \\
\quad \langle \mathsf{receive}\,() \to r \rangle \mapsto \mathsf{copipe}\, r\, p & \quad \langle \mathsf{send}\, n \to r \rangle \mapsto \mathsf{pipe}\, r\,(\lambda().c\, n)
\end{array}$$

$$\begin{array}{rl}
\mathsf{pipe}\,(\lambda().\mathsf{nats}\, 0)\, \mathsf{grabANat} & \leadsto^{+} \mathsf{copipe}\,(\lambda x.x)\,(\lambda().\mathsf{nats}\, 0) \\
& \leadsto^{+} \mathsf{pipe}\,(\lambda().\mathsf{nats}\, 1)\,(\lambda().0) \leadsto^{+} 0
\end{array}$$

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\quad \mathbf{return}\, x \mapsto x & \quad \mathbf{return}\, x \mapsto x \\
\quad \langle \mathsf{receive}\,() \to r \rangle \mapsto \mathsf{copipe}\, r\, p & \quad \langle \mathsf{send}\, n \to r \rangle \mapsto \mathsf{pipe}\, r\,(\lambda().c\, n)
\end{array}$$

$$\begin{aligned}
\mathsf{pipe}\,(\lambda().\mathsf{nats}\, 0)\,\mathsf{grabANat} &\leadsto^{+} \mathsf{copipe}\,(\lambda x.x)\,(\lambda().\mathsf{nats}\, 0) \\
&\leadsto^{+} \mathsf{pipe}\,(\lambda().\mathsf{nats}\, 1)\,(\lambda().0) \leadsto^{+} 0
\end{aligned}$$

Exercise: implement pipes using deep handlers

Small-step operational semantics for shallow effect handlers

Reduction rules

$$\begin{aligned}
\textbf{let } x = V \textbf{ in } N &\leadsto N[V/x] \\
\textbf{handle}^\dagger\ V \textbf{ with } H &\leadsto N_{\mathrm{ret}}[V/x] \\
\textbf{handle}^\dagger\ \mathcal{E}[\mathsf{op}\ V] \textbf{ with } H &\leadsto N_{\mathsf{op}}[V/p, (\lambda x.\mathcal{E}[x])/r], \quad \mathsf{op} \# \mathcal{E}
\end{aligned}$$

$$\begin{aligned}
\text{where } H = \textbf{return}\, x \quad &\mapsto N_{\mathrm{ret}} \\
\langle \mathsf{op}_1\ p \to r \rangle \quad &\mapsto N_{\mathsf{op}_1} \\
&\dots \\
\langle \mathsf{op}_k\ p \to r \rangle \quad &\mapsto N_{\mathsf{op}_k}
\end{aligned}$$

Evaluation contexts

$$\mathcal{E} ::= [\ ] \mid \textbf{let } x = \mathcal{E} \textbf{ in } N \mid \textbf{handle}^\dagger\ \mathcal{E} \textbf{ with } H$$

Small-step operational semantics for shallow effect handlers

Reduction rules

$$\begin{array}{rcl}
\textbf{let}\ x = V\ \textbf{in}\ N & \rightsquigarrow & N[V/x] \\
\textbf{handle}^{\dagger}\ V\ \textbf{with}\ H & \rightsquigarrow & N_{\mathrm{ret}}[V/x] \\
\textbf{handle}^{\dagger}\ \mathcal{E}[\mathsf{op}\ V]\ \textbf{with}\ H & \rightsquigarrow & N_{\mathsf{op}}[V/p, (\lambda x.\mathcal{E}[x])/r], \quad \mathsf{op} \# \mathcal{E}
\end{array}$$

$$\begin{array}{rrcl}
\text{where}\ H = & \textbf{return}\ x & \mapsto & N_{\mathrm{ret}} \\
& \langle \mathsf{op}_1\ p \to r\rangle & \mapsto & N_{\mathsf{op}_1} \\
& & \ldots & \\
& \langle \mathsf{op}_k\ p \to r\rangle & \mapsto & N_{\mathsf{op}_k}
\end{array}$$

Evaluation contexts

$$\mathcal{E} ::= [\ ] \mid \textbf{let}\ x = \mathcal{E}\ \textbf{in}\ N \mid \textbf{handle}^{\dagger}\ \mathcal{E}\ \textbf{with}\ H$$

Exercise: express shallow handlers as deep handlers

Built-in effects

Console I/O

$$\begin{aligned}\mathsf{Console} = \{\mathsf{inch} &: \quad 1 \Rightarrow \mathsf{char} \\ \mathsf{ouch} &: \mathsf{char} \Rightarrow 1\}\end{aligned}$$

$$\mathsf{print}\, s = \mathsf{map}\,(\lambda c.\mathsf{ouch}\; c)\; s; ()$$

State

$$\begin{aligned}\mathsf{State} = \{\mathsf{new} &: \; a. \qquad\qquad a \Rightarrow \mathsf{Ref}\; a, \\ \mathsf{write} &: a.\; (\mathsf{Ref}\; a \times a) \Rightarrow 1, \\ \mathsf{read} &: \; a. \qquad \mathsf{Ref}\; a \Rightarrow a\}\end{aligned}$$

Example 5: actors

Process ids

$$\mathsf{Pid}\ a = \mathsf{Ref}\ (\mathsf{List}\ a)$$

Effect signature

$$\begin{aligned}\mathsf{Actor}\ a = \{\mathsf{self} &: && 1 \Rightarrow \mathsf{Pid}\ a,\\ \mathsf{spawn} &: b.\ && (1 \to [\mathsf{Actor}\ b]1) \Rightarrow \mathsf{Pid}\ b,\\ \mathsf{send} &: b.\ && (b \times \mathsf{Pid}\ b) \Rightarrow 1,\\ \mathsf{recv} &: && 1 \Rightarrow a\}\end{aligned}$$

Example 5: actors

Process ids

$$\mathsf{Pid}\ a = \mathsf{Ref}\ (\mathsf{List}\ a)$$

Effect signature

$$\begin{aligned}\mathsf{Actor}\ a = \{&\mathsf{self} : && 1 \Rightarrow \mathsf{Pid}\ a,\\ &\mathsf{spawn} : b.\ (1 \to [\mathsf{Actor}\ b]1) && \Rightarrow \mathsf{Pid}\ b,\\ &\mathsf{send} : b.\ (b \times \mathsf{Pid}\ b) && \Rightarrow 1,\\ &\mathsf{recv} : && 1 \Rightarrow a\}\end{aligned}$$

An actor chain

$$\begin{aligned}&\mathsf{spawnMany}\ p\ 0 = \mathsf{send}\ (\text{“ping!”}, p)\\ &\mathsf{spawnMany}\ p\ n = \mathsf{spawnMany}\ (\mathsf{spawn}\ (\lambda().\mathbf{let}\ s = \mathsf{recv}\ ()\ \mathbf{in}\ \mathsf{print}\ \text{“.”}; \mathsf{send}\ (s, p)))\ (n-1)\end{aligned}$$

$$\mathsf{chain}\ n = \mathsf{spawnMany}\ (\mathsf{self}\ ())\ n; \mathbf{let}\ s = \mathsf{recv}\ ()\ \mathbf{in}\ \mathsf{print}\ s$$

Example 5: actors

Actors via cooperative concurrency

$$
\begin{array}{l}
\mathsf{act}\ \mathit{mine} = \\
\quad \begin{array}{ll}
\textbf{return}\,() & \mapsto () \\
\langle \mathsf{self}\,() \to r \rangle & \mapsto r\ \mathit{mine}\ \mathit{mine} \\
\langle \mathsf{spawn}\ \mathit{you} \to r \rangle & \mapsto \textbf{let}\ \mathit{yours} = \mathsf{new}\,[]\ \textbf{in} \\
 & \ \mathsf{fork}\,(\lambda().\mathsf{act}\ \mathit{yours}\,(\mathit{you}\,()));\ r\ \mathit{mine}\ \mathit{yours} \\
\langle \mathsf{send}\,(m, \mathit{yours}) \to r \rangle & \mapsto \textbf{let}\ \mathit{ms} = \mathsf{read}\ \mathit{yours}\ \textbf{in} \\
 & \ \mathsf{write}\,(\mathit{yours}, \mathit{ms} +\!\!+ [m]);\ r\ \mathit{mine}\,() \\
\langle \mathsf{recv}\,() \to r \rangle & \mapsto \textbf{case}\ \mathsf{read}\ \mathit{mine}\ \textbf{of} \\
 & \qquad \begin{array}{ll}
 [] & \mapsto \mathsf{yield}\,();\ r\ \mathit{mine}\,(\mathsf{recv}\,()) \\
 (m :: \mathit{ms}) & \mapsto \mathsf{write}\,(\mathit{mine}, \mathit{ms});\ r\ \mathit{mine}\ m
 \end{array}
\end{array}
\end{array}
$$

Example 5: actors

Actors via cooperative concurrency

$$
\begin{array}{l}
\mathsf{act}\ \mathit{mine} = \\
\quad \begin{array}{ll}
\mathbf{return}\,() & \mapsto () \\
\langle \mathsf{self}\,() \to r\rangle & \mapsto r\ \mathit{mine}\ \mathit{mine} \\
\langle \mathsf{spawn}\ \mathit{you} \to r\rangle & \mapsto \mathbf{let}\ \mathit{yours} = \mathsf{new}\,[]\ \mathbf{in} \\
 & \quad\ \mathsf{fork}\,(\lambda().\mathsf{act}\ \mathit{yours}\,(\mathit{you}\,()));\ r\ \mathit{mine}\ \mathit{yours} \\
\langle \mathsf{send}\,(m, \mathit{yours}) \to r\rangle & \mapsto \mathbf{let}\ \mathit{ms} = \mathsf{read}\ \mathit{yours}\ \mathbf{in} \\
 & \quad\ \mathsf{write}\,(\mathit{yours}, \mathit{ms} +\!\!+ [m]);\ r\ \mathit{mine}\,() \\
\langle \mathsf{recv}\,() \to r\rangle & \mapsto \mathbf{case}\ \mathsf{read}\ \mathit{mine}\ \mathbf{of} \\
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\end{array}
\end{array}
$$

$$\mathsf{cooperate}\,[\mathbf{handle}\ \mathsf{chain}\ 64\ \mathbf{with}\ \mathsf{act}\,(\mathsf{new}\,[])] \Longrightarrow ()$$

....ping!

Effect handler oriented programming languages

Eff	https://www.eff-lang.org/
Frank	https://github.com/frank-lang/frank
Helium	https://bitbucket.org/pl-uwr/helium
Links	https://www.links-lang.org/
Koka	https://github.com/koka-lang/koka
Multicore OCaml	https://github.com/ocamllabs/ocaml-multicore/wiki

Effect handlers — some of my contributions

Handlers in action (ICFP 2013)
with Kammar and Oury

Effect handlers in Links (TyDe 2016 / JFP 2020)
with Hillerström

Frank programming language (POPL 2017 / JFP 2020)
with Convent, McBride, and McLaughlin

Expressive power of effect handlers (ICFP 2017 / JFP 2019)
with Forster, Kammar, and Pretnar

Continuation-passing style for effect handlers (FSCD 2017 / JFP 2020)
with Atkey, Hillerström, and Sivaramakrishnan

Shallow effect handlers (APLAS 2018 / JFP 2020)
with Hillerström

Linear effect handlers for session exceptions (POPL 2019)
with Decova, Fowler, and Morris

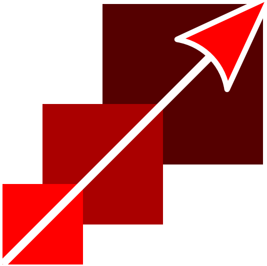
Scalability challenges

Modularity — effect typing

- Effect encapsulation
- Linearity
- Generativity
- Indexed effects
- Equations

Efficiency — compilation techniques

- Segmented stacks (Multicore OCaml / C library)
- Continuation Passing Style (JavaScript backends)
- Fusion (Haskell libraries / Eff)
- Staging (Scala Effekt library)



New directions

Effect handlers for Wasm
add effect handlers once and for all — avoid pitfalls of JavaScript

Asynchronous effects
pre-emptive concurrency; reactive programming

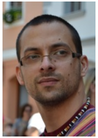
Gradually typed effect handlers
transition mainstream languages towards effect typing

Hardware capabilities as dynamic effects
safe effect handlers in C? efficient implementation?

Lexically scoped effect handlers
improved hygiene? improved performance? improved reasoning?

Resources

Jeremy Yallop's effects bibliography
https://github.com/yallop/effects-bibliography



Matija Pretnar's tutorial
"An introduction to algebraic effects and handlers",
MFPS 2015

Andrej Bauer's tutorial
"What is algebraic about algebraic effects and handlers?",
Dagstuhl and OPLSS 2018

Part III

Bonus slides

Example 6: effect pollution

Effect signatures

$$\text{Receiver} = \{\text{receive} : 1 \Rightarrow \text{Nat}\} \qquad \text{Failure} = \{\text{fail} : a.1 \Rightarrow a\}$$

Example 6: effect pollution

Effect signatures

$$\mathsf{Receiver} = \{\mathsf{receive} : 1 \Rightarrow \mathsf{Nat}\} \qquad \mathsf{Failure} = \{\mathsf{fail} : a.1 \Rightarrow a\}$$

Handlers

$$\begin{array}{l} \mathsf{receives}\,([]) = \\ \quad \textbf{return}\, x \qquad \mapsto x \\ \quad \langle \mathsf{receive}\,() \to r \rangle \mapsto \mathsf{fail}\,() \end{array} \qquad \begin{array}{l} \mathsf{receives}\,(n :: ns) = \\ \quad \textbf{return}\, x \qquad \mapsto x \\ \quad \langle \mathsf{receive}\,() \to r \rangle \mapsto r\; ns\; n \end{array}$$

$$\begin{array}{l} \mathsf{maybeFail} = \\ \quad \textbf{return}\, x \qquad \mapsto \mathsf{Just}\, x \\ \quad \langle \mathsf{fail}\,() \to r \rangle \mapsto \mathsf{Nothing} \end{array}$$

Example 6: effect pollution

Effect signatures

$$\text{Receiver} = \{\text{receive} : 1 \Rightarrow \text{Nat}\} \qquad \text{Failure} = \{\text{fail} : a.1 \Rightarrow a\}$$

Handlers

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$$\begin{aligned} &\text{maybeFail} = \\ &\quad \textbf{return}\, x \mapsto \text{Just}\, x \\ &\quad \langle \text{fail}\,() \to r\rangle \mapsto \text{Nothing} \end{aligned}$$

$$\text{bad}\; ns\; t = \textbf{handle}\; (\textbf{handle}\; t\,()\; \textbf{with}\; \text{receives}\; ns)\; \textbf{with}\; \text{maybeFail}$$

Example 6: effect pollution

Effect signatures

$$\mathsf{Receiver} = \{\mathsf{receive} : 1 \Rightarrow \mathsf{Nat}\} \qquad \mathsf{Failure} = \{\mathsf{fail} : a.1 \Rightarrow a\}$$

Handlers

$$\begin{array}{ll} \mathsf{receives}\,([]) = & \\ \quad \mathbf{return}\, x & \mapsto x \\ \quad \langle \mathsf{receive}\,() \to r \rangle & \mapsto \mathsf{fail}\,() \end{array} \qquad \begin{array}{ll} \mathsf{receives}\,(n :: ns) = & \\ \quad \mathbf{return}\, x & \mapsto x \\ \quad \langle \mathsf{receive}\,() \to r \rangle & \mapsto r\; ns\; n \end{array}$$

$$\begin{array}{ll} \mathsf{maybeFail} = & \\ \quad \mathbf{return}\, x & \mapsto \mathsf{Just}\, x \\ \quad \langle \mathsf{fail}\,() \to r \rangle & \mapsto \mathsf{Nothing} \end{array}$$

$$\mathsf{bad}\; ns\; t = \mathbf{handle}\; (\mathbf{handle}\; t\,()\; \mathbf{with}\; \mathsf{receives}\; ns)\; \mathbf{with}\; \mathsf{maybeFail}$$

$$\mathsf{bad}\,[1, 2]\,(\lambda().\mathsf{receive}\,() + \mathsf{fail}\,()) \Longrightarrow \mathsf{Nothing}$$

Example 7: counting

Predicates as higher order functions

$$\mathsf{Pred} = (\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool}$$

Signature of a counting function

$$\mathsf{count} : ((\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool}) \to \mathsf{Nat}$$

Exclusive or

$$\mathsf{count}\,(\lambda v.\textbf{if}\ v\ 0\ \textbf{then}\ \mathsf{not}\,(v\ 1)\ \textbf{else}\ v\ 1) = 2$$

Example 7: counting

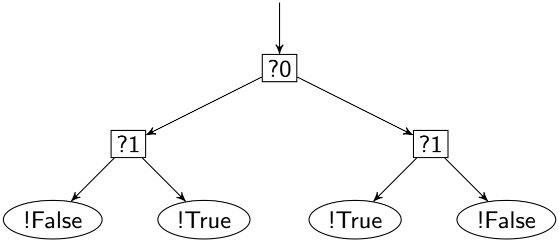
Counting with a choice handler

$$\mathsf{count} : ((\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool}) \to \mathsf{Nat}$$

$$\begin{aligned}\mathsf{count} = \lambda p.\mathbf{handle}\ p\,(\lambda\_.\mathsf{choose}\,())\ \mathbf{with}\\ \mathbf{return}\ x \quad &\mapsto \mathbf{if}\ x\ \mathbf{then}\ 1\ \mathbf{else}\ 0\\ \langle \mathsf{choose}\ () \to r\rangle \quad &\mapsto r\,\mathsf{True} + r\,\mathsf{False}\end{aligned}$$

Exclusive or

$$\mathsf{count}\,(\lambda v.\mathbf{if}\ v\ 0\ \mathbf{then}\ \mathsf{not}\,(v\ 1)\ \mathbf{else}\ v\ 1)$$



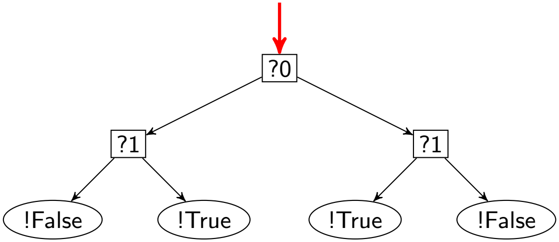
Example 7: counting

Counting with a choice handler

$$\begin{aligned}
&\mathsf{count} : ((\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool}) \to \mathsf{Nat}\\
&\mathsf{count} = \lambda p.\mathbf{handle}\ p\,(\lambda_.\mathsf{choose}\,())\ \mathbf{with}\\
&\qquad\qquad \mathbf{return}\ x \qquad\qquad \mapsto \mathbf{if}\ x\ \mathbf{then}\ 1\ \mathbf{else}\ 0\\
&\qquad\qquad \langle \mathsf{choose}\ () \to r\rangle \ \mapsto r\,\mathsf{True} + r\,\mathsf{False}
\end{aligned}$$

Exclusive or

$$\mathsf{count}\,(\lambda v.\mathbf{if}\ v\ 0\ \mathbf{then}\ \mathsf{not}\,(v\ 1)\ \mathbf{else}\ v\ 1)$$



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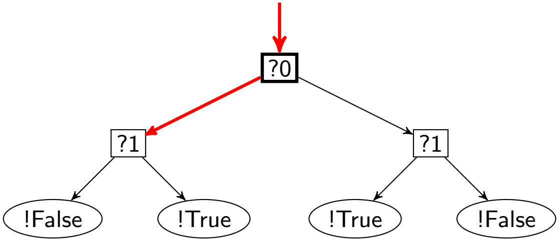
Exclusive or

$$\text{count}\,(\lambda v.\textbf{if}\ v\ 0\ \textbf{then}\ \text{not}\,(v\ 1)\ \textbf{else}\ v\ 1)$$



Example 7: counting

Counting with a choice handler

$$\begin{aligned}
&\mathsf{count} : ((\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool}) \to \mathsf{Nat} \\
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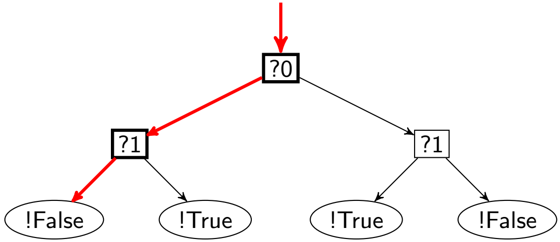

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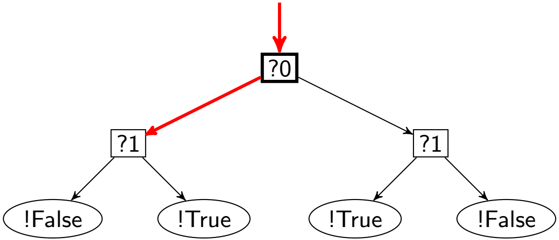

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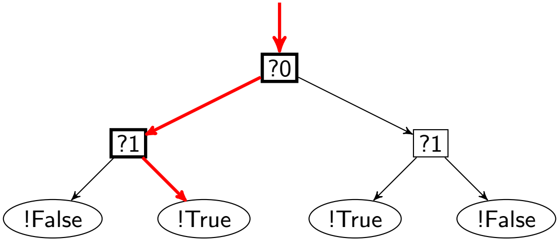

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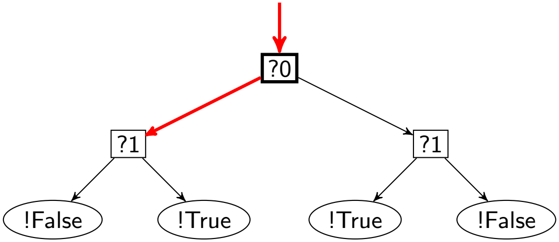

Example 7: counting

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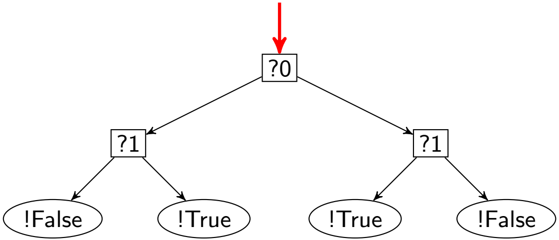

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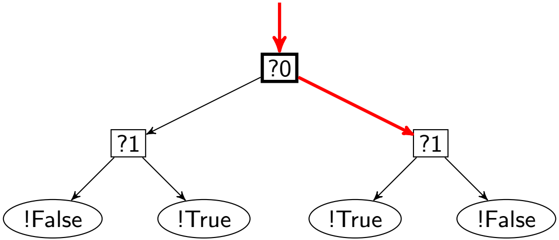

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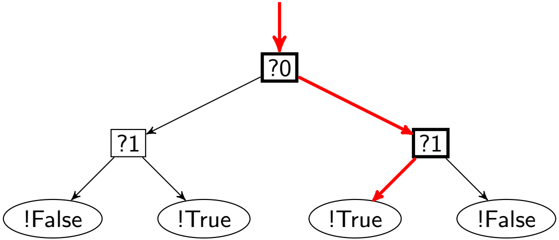

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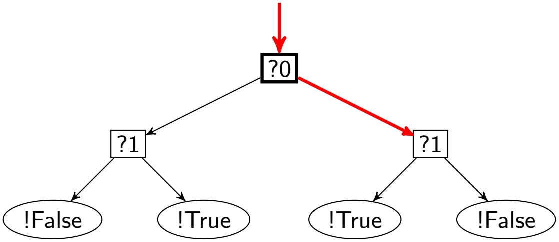

Example 7: counting

Counting with a choice handler

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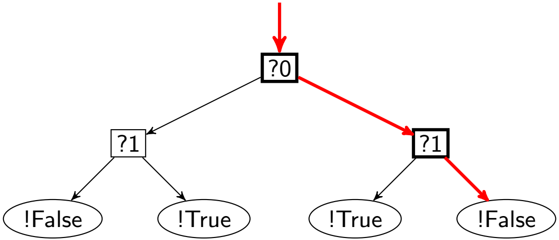

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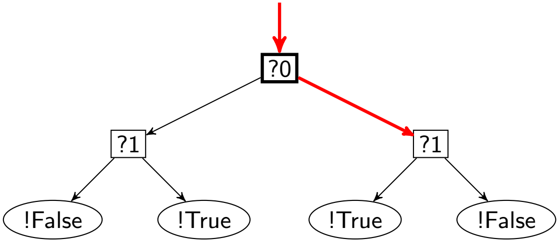

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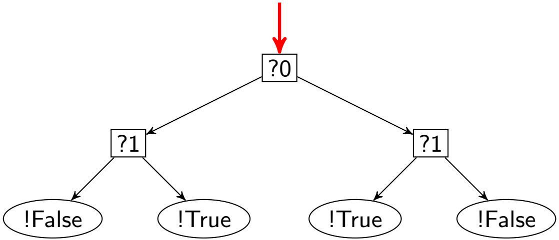

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