

# Behavioural Type-Based Static Verification Framework

for

# GO



Juhen Lange



Nicholas Ng



Bernardo Toninho



Nobuko Yoshida



# ETAPS 2019 TEST-OF-TIME AWARD

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[Language primitives and type discipline for structured communication-based programming](#) by Kohei Honda, Vasco T. Vasconcelos and Makoto Kubo

# POPL 2008 MOST INFLUENTIAL PAPER AWARD



POPL 2008 Most Influential Paper Award

Kohei Honda, Nobuko Yoshida and Marco Carbone

Multiparty asynchronous session types







# LICS 2018 TEST OF TIME AWARD

LICS'98

## A fully abstract game semantics for general references

Samson Abramsky      Kohei Honda

LFCS, University of Edinburgh

{samson, kohei}@dcs.ed.ac.uk

Guy McCusker

St John's College, Oxford

mccusker@comlab.ox.ac.uk

### Abstract

*A games model of a programming language with higher-order store in the style of ML-references is introduced. The category used for the model is obtained by relaxing certain behavioural conditions on a category of games previously used to provide fully abstract models of pure functional languages. The model is shown to be fully abstract by means of factorization arguments which reduce the question of definability for the language with higher-order store to that for its purely functional fragment.*



<http://mrg.doc.ic.ac.uk>

# Mobility Research Group



$\pi$ -calculus, Session Types research at Imperial College

Home

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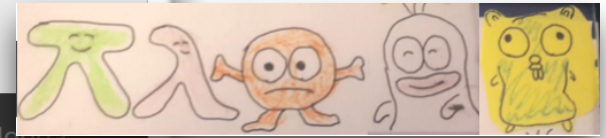
Talks

Tutorials

Tools

Awards

Kohei Honda



## NEWS

The paper *Multiparty asynchronous session types* by Kohei Honda, Nobuko Yoshida, and Marco Carbone, published in POPL 2008 has been awarded the ACM SIGPLAN Most Influential POPL Paper Award today at POPL 2018.

» more

10 Jan 2018

Estafet has published a page on their usage of the Scribble language developed in our group with RedHat and other industry partners.

» more

25 Sep 2017

Nick spoke at Golang UK 2017 on applying behavioural types to verifv concurrent Go programs.

## SELECTED PUBLICATIONS

2018

Julien Lange , Nicholas Ng , Bernardo Toninho , Nobuko Yoshida : [A Static Verification Framework for Message Passing in Go using Behavioural Types](#). *To appear in ICSE 2018* .

Bernardo Toninho , Nobuko Yoshida : [Depending On Session Typed Process](#). *To appear in FoSSaCS 2018* .

Bernardo Toninho , Nobuko Yoshida : [On Polymorphic Sessions And Functions: A Talk of Two \(Fully Abstract\) Encodings](#). *To appear in ESOP 2018* .

Rumyana Neykova , Raymond Hu , Nobuko Yoshida , Fahd Abdeljallal : [Session Type Providers: Compile-time API Generation for Distributed Protocols with Interaction Refinements in F#](#). *To appear in CC 2018* .

### Post-docs:

Simon CASTELLAN

David CASTRO

Francisco FERREIRA

Raymond HU

Rumyana NEYKOVA

Nicholas NG

Alceste SCALAS

### PhD Students:

Assel ALTAYEVA

Juliana FRANCO

Eva GRAVERSEN



## Scribble: Describing Multi Party Protocols

Scribble is a language to describe application-level protocols among communicating systems. A protocol represents an agreement on how participating systems interact with each other. Without a protocol, it is hard to do meaningful interaction: participants simply cannot communicate effectively, since they do not know when to expect the other parties to send data, or whether the other party is ready to receive data. However, having a description of a protocol has further benefits. It enables verification to ensure that the protocol can be implemented without resulting in unintended consequences, such as deadlocks.

### Describe

Scribble is a language for describing multiparty protocols from a global, or endpoint neutral, perspective.

### Verify

Scribble has a theoretical foundation, based on the Pi Calculus and Session Types, to ensure that protocols described using the language are sound, and do not suffer from deadlocks or livelocks.

### Project

Endpoint projection is the term used for identifying the responsibility of a particular role (or endpoint) within a protocol.

### Implement

Various options exist, including (a) using the endpoint projection for a role to generate a skeleton code, (b) using session type APIs to clearly describe the behaviour, and (c) statically verify the code against the projection.

### Monitor

Use the endpoint projection for roles defined within a Scribble protocol, to monitor the activity of a particular endpoint, to ensure it correctly implements the expected behaviour.

# OOI Collaboration

**OOI OCEAN OBSERVING INITIATIVE**

Location

SEARCH

RESOURCES

- All Resources
- Data Products
- Observations
- Platforms
- Instruments

Dashboard

RECENT IMAGES

- Glider
- Georgian Coral
- Acoustic Release
- Seafloor CDP
- Marine Laptop
- Surface Buoy

POPULAR RESOURCES

RECENT UPDATES

DATE	TITLE	STATUS	DESCRIPTION
2012-02-02 22:00:00	Oregon Coast Wave Height	Open	Observation data base
2012-02-02 22:00:00	Water Surface Elevations	Open	Observation data base
2012-02-02 22:00:00	Oregon Coast Wave Height	Open	Observation data base
2012-02-02 22:00:00	Water Surface Elevations	Open	Observation data base
2012-02-02 22:00:00	Oregon Coast Wave Height	Open	Observation data base
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2012-02-02 22:00:00	Oregon Coast Wave Height	Open	Observation data base
2012-02-02 22:00:00	Water Surface Elevations	Open	Observation data base

- **TCS'16:** Monitoring Networks through Multiparty Session Types. Laura Bocchi , Tzu-Chun Chen , Romain Demangeon , Kohei Honda , Nobuko Yoshida
- **LMCS'16:** Multiparty Session Actors. Rumyana Neykova, Nobuko Yoshida
- **FMSD'15:** Practical interruptible conversations: Distributed dynamic verification with multiparty session types and Python. Romain Demangeon , Kohei Honda , Raymond Hu , Rumyana Neykova , Nobuko Yoshida
- **TGC'13:** The Scribble Protocol Language. Nobuko Yoshida , Raymond Hu , Rumyana Neykova , Nicholas Ng



# End-to-End Switching Programme by DCC



**Estafet**

Innovate | Deliver | Transform

1. All design work takes place in ABACUS, DCC's enterprise architecture tool. This can export standard XMI files (an open standard for UML5)

2. XMI is converted into OpenTracing format for consumption by managed service



OPENTRACING



3. OpenTracing files are combined to build a model in Scribble

4. Model holds *types* rather than *instances* to understand behaviour

5. Scribble compiler identifies inconsistency, change & design flaws

6. Issues highlighted graphically in Eclipse

7. Generate exception report and send back to DCC



# Interactions with Industries

## #unctional Londoners Meetup Group

CC'18

6 days ago · 6:30 PM

### Session Types with Fahd Abdeljallal



43 Members

Synopsis: Session types are a formalism to codify the structure of a communication, using types to specify the communication protocol used. This formalism provides the... [LEARN MORE](#)

ECOOP'17

## Distributed Systems vs. Compositionality

Dr. Roland Kuhn  
@rolandkuhn — CTO of Actyx

actyx

### Current State

- behaviors can be composed both sequentially and concurrently
- effects are not yet tracked
- Scribble generator for Scala not yet there
- theoretical work at Imperial College, London (Prof. Nobuko Yoshida & Alceste Scalas)

ECOOP'16

CC'18

## A Session Type Provider

Compile-Time API Generation of Distributed Protocols with Refinements in F#

Rumyana Neykova  
Imperial College London  
United Kingdom

Raymond Hu  
Imperial College London  
United Kingdom

Nobuko Yoshida  
Imperial College London  
United Kingdom

Fahd Abdeljallal  
Imperial College London  
United Kingdom

### Abstract

We present a library for the specification and implementation of distributed protocols in native F# (and other .NET languages) based on multiparty session types (MPST). There are two main contributions. Our library is the first practical development of MPST to support what we refer to as *interaction refinements*: a collection of features related to the refinement of *protocols*, such as message-type refinements (value constraints) and message-value dependent control flow. A well-typed endpoint program using our library is guaranteed to perform only compliant session I/O actions on the refined protocol, up to premature termination. Our library is developed as a session *type provider*,

### 1 Introduction

*Type providers* [20, 27] are a .NET feature for a form of compile-time meta programming, designed to bridge between programming in statically typed languages such as F# and C#, and working with so-called *information spaces*—structured data sources such as SQL databases or XML data.

A type provider works as a compiler plugin that performs on-demand generation of *types*: it takes a schema for an external information space, and generates types that allow the data to be manipulated via a strongly-typed interface, with benefits such as static error detection and IDE auto-completion. For example, an instantiation of the in-built type provider for WSDL Web services [6] may look like



Graydon Hoare

@graydon\_pub

(This stuff is `_fantastic_`)

11:31 PM · 11 Mar 2018

32 Retweets 83 Likes



shots fired @zeeshanlakhani · Mar 12

Replying to @graydon\_pub @dsyme

Awesome!

Brendan Zabarauskas @brendanzab ·

Replying to @graydon\_pub

This stuff fills me with hope!

Ryan Riley @panesofglass · Mar 12

Replying to @graydon\_pub

This is amazing! I guess I need to switch





## Selected Publications [2018-2019]

- [PLDI19] [Alceste Scalas](#), NY, Elias Benussi: [Verifying message-passing programs with dependent behavioural types](#).
- [CAV19] [Julien Lange](#), NY: [Verifying Asynchronous Interactions via Communicating Session Automata](#).
- [CONCUR19] Mario Bravetti, Marco Carbone, [Julien Lange](#), NY, Gianluigi Zavattaro: [A Sound Algorithm for Asynchronous Session Subtyping](#)
- [FSE19] Nicola Atzei, Massimo Bartoletti, Stefano Lande, NY, Roberto Zunino: [Developing secure Bitcoin contracts with BitML](#)
- [ECOOP19] Rupak Majumdar, Marcus Pirron, NY, and Damien Zufferey, Motion Session Types for Robotic Interactions
- [FoSSaCs19] [Simon Castellan](#), NY: [Causality in Linear Logic](#)
- [ESOP19] [Laura Bocchi](#), Maurizio Murgia, Vasco T. Vasconcelos, NY: [Asynchronous Timed Session Types](#)
- [POPL19] [Simon Castellan](#), NY: [Two Sides of the Same Coin: Session Types and Game Semantics](#)
- [POPL19] [David Castro](#), [Raymond Hu](#), Sung-Shik Jongmans, [Nicholas Ng](#), NY: [Distributed Programming Using Role Parametric Session Types in Go](#)
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- [POPL19] [Simon Castellan](#), NY: [Two Sides of the Same Coin: Session Types and Game Semantics](#)
- [POPL19] [David Castro](#), [Raymond Hu](#), [Sung-Shik Jongmans](#), [Nicholas Ng](#), NY: [Distributed Programming Using Role Parametric Session Types in Go](#)
- [POPL19] [Alceste Scalas](#), [Nobuko Yoshida](#): [Less Is More: Multiparty Session Types Revisited](#)
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- [ICSE18] [Julien Lange](#), [Nicholas Ng](#), [Bernardo Toninho](#), NY: [A Static Verification Framework for Message Passing in Go using Behavioural Types](#)
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- [ESOP18] [Bernardo Toninho](#), NY: [On Polymorphic Sessions And Functions: A Tale of Two \(Fully Abstract\) Encodings](#)
- [CC18] [Rumyana Neykova](#), [Raymond Hu](#), NY, [Fahd Abdeljallal](#): [A Session Type Provider: Compile-time API Generation for Distributed Protocols with Interaction Refinements in F#](#)

# Behavioural Type-Based Static Verification Framework

for

# GO



Juhen Lange



Nicholas Ng



Bernardo  
Toninho



Nobuko  
Yoshida





# GOLANG UK CONFERENCE

16<sup>th</sup>\*, 17<sup>th</sup> & 18<sup>th</sup> AUGUST 2017

The Brewery, London



Imperial College  
London

Home College and Campus Science **Engineering** Health Business Search here... Go

## Go concurrency verification research at DoC grabs headline

POPL'17

### the morning paper

ICSE'18

an interesting/influential/important paper from the world of CS every weekday morning, as selected by Adrian Colyer

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#### A static verification framework for message passing in Go using behavioural types

JANUARY 25, 2018

tags: Concurrency, Programming Languages

##### A static verification framework for message passing in Go using behavioural types

*With thanks to Alexis Richardson who first forwarded this paper to me.*

We're jumping ahead to ICSE 18 now, and a paper that has been accepted for publication there later this year. It fits with the theme we've been exploring this week though, so I thought I'd cover it now. We've seen verification techniques applied in the context of **Rust** and **JavaScript**, looked at the integration of **linear types in Haskell**, and today it is the turn of Go!

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L (Principles

# GO

programming language @ Google (2009)

- ▶ **Message - Passing** based multicore PL, successor of C
- ▶ *Do not communicate by shared memory;*  
instead, share memory by *communicating*

Go Lang Proverb

- ▶ **Explicit channel-based concurrency**
  - Buffered I/O communication channels
  - Lightweight thread spawning - **goroutines**
  - Selective **send/receive**

CSP<sub>80'</sub>

# FUN

Dropbox, Netflix, Docker, CoreOS

- ▶ **GO** has a runtime deadlock detector
- ▶ How can we detect *partial deadlock* and *channel errors* for realistic programs?
- ▶ Use *behavioural types* in process calculi  
e.g. [ACM Survey, 2016] **185** citations, 6 pages
- ▶ Dynamic channel creations, unbounded thread creations, recursions, ..
- ▶ **Scalable** (synchronous/asynchronous) **Modular, Refinable**

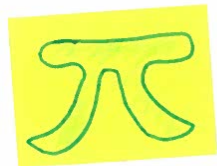


▶  has a runtime deadlock detector

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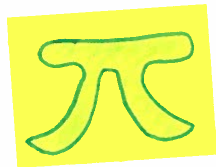
▶ **Scalable** (synchronous/asynchronous) **Modular, Refinable**



▶  has a runtime deadlock detector

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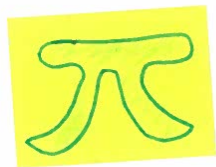
- ▶ Dyn.   , unbounded thread creations, recursions, ..
- ▶ **Scalable** (synchronous/asynchronous) **Modular, Refinable**

▶  has a runtime deadlock detector

▶ How can we detect *partial deadlock* and *channel errors* for realistic programs?

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e.g. [ACM Survey, 2016] **185** citations, 6 pages



**186 ??**

▶ channel creations, unbounded thread creations, recursions, ..

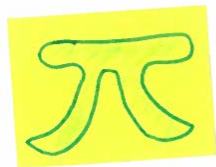
▶ **Scalable** (synchronous/asynchronous) **Modular, Refinable**

▶  has a runtime deadlock detector

▶ How can we detect *partial deadlock* and *channel errors* for realistic programs?

▶ Use *behavioural types* in process calculi

e.g. [ACM Survey, 2016] **185** citations, 6 pages



▶ Dynamic channel creations, unbounded thread creation, ...



▶ **Scalable** (synchronous/asynchronous) **Modular**, **retinable**

**Understandable**

# Our Framework

## STEP 1 Extract Behavioural Types

- ▶ (Most) Message passing features of GO
- ▶ Tricky primitives : selection, channel creation

## STEP 2 Check Safety/Liveness of Behavioural Types

- ▶ Model-Checking (Finite Control)

## STEP 3

- ▶ Relate Safety/Liveness of Behavioural Types and GO Programs
  - ▶ 3 Classes [POPL'17]
  - ▶ Termination Check



# Our Framework

## STEP 1

Extract Behavioural Types

- ▶ (Most) Message passing features of GO
- ▶ Tricky primitives: selection, channel creation

## STEP 2

Check Safety/Liveness of Behavioural Types

- ▶ Model-Checking (Finite Control)

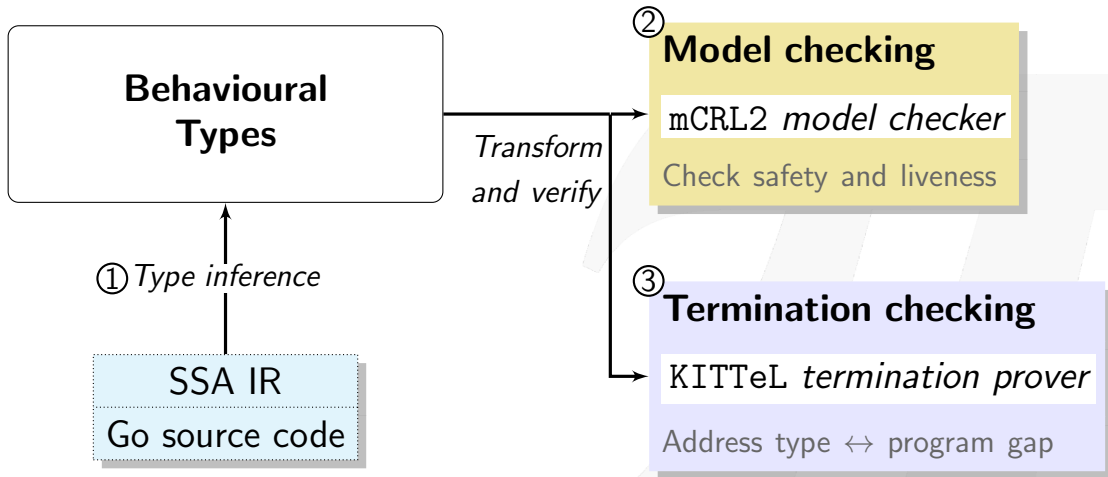
## STEP 3

- ▶ Relate Safety/Liveness of Behavioural Types and GO Programs
- ▶ 3 Classes [POPL'17]
- ▶ Termination Check



# Static verification framework for Go

## Overview



# Concurrency in Go

## Goroutines

```
1 func main() {  
2     ch := make(chan string)  
3     go send(ch)  
4     print(<-ch)  
5     close(ch)  
6 }  
7  
8 func send(ch chan string) {  
9     ch <- "Hello Kent!"  
10 }
```

go keyword + function call

- Spawns function as goroutine
- Runs in parallel to parent

# Concurrency in Go

## Channels

```
1 func main() {
2   ch := make(chan string)
3   go send(ch)
4   print(<-ch)
5   close(ch)
6 }
7
8 func send(ch chan string) {
9   ch <- "Hello Kent!"
10 }
```

Create **new** channel

- Synchronous by default

Receive from channel

Close a channel

- No more values sent to it
- Can only close once

Send to channel

# Concurrency in Go

## Channels

```
1 func main() {
2     ch := make(chan string)
3     go send(ch)
4     print(<-ch)
5     close(ch)
6 }
7
8 func send(ch chan string) {
9     ch <- "Hello Kent!"
10 }
```

Also `select-case`:

- Wait on multiple channel operations
- `switch-case` for communication



# Concurrency in Go

## Deadlock detection

```
1 func main() {
2     ch := make(chan string)
3     go send(ch)
4     print(<-ch)
5     close(ch)
6 }
7
8 func send(ch chan string) {
9     ch <- "Hello Kent!"
10 }
```

- Send message thru channel
- Print message on screen

Output:

```
$ go run hello.go
Hello Kent!
$
```

# Concurrency in Go

## Deadlock detection

Missing 'go' keyword

```

1 // import _ "net"
2 func main() {
3     ch := make(chan string)
4     send(ch) // Oops
5     print(<-ch)
6     close(ch)
7 }
8
9 func send(ch chan string) {
10     ch <- "Hello Kent!"
11 }

```

- Only one (main) goroutine
- Send without receive - blocks

Output:

```

$ go run deadlock.go
fatal error: all goroutines
are asleep - deadlock!
$

```

# Concurrency in Go

## Deadlock detection

Missing 'go' keyword

```

1 // import _ "net"
2 func main() {
3     ch := make(chan string)
4     send(ch) // Oops
5     print(<-ch)
6     close(ch)
7 }
8
9 func send(ch chan string) {
10     ch <- "Hello Kent!"
11 }

```

### Go's runtime deadlock detector

- Checks if **all** goroutines are blocked ('global' deadlock)
- Print message then crash
- Some packages disable it (e.g. net)

# Concurrency in Go 🐼

## Deadlock detection

Missing 'go' keyword

```
1 import _ "net" // unused
2 func main() {
3     ch := make(chan string)
4     send(ch) // Oops
5     print(<-ch)
6     close(ch)
7 }
8
9 func send(ch chan string) {
10     ch <- "Hello Kent"
11 }
```

Import unused, unrelated package

# Concurrency in Go

## Deadlock detection

Missing 'go' keyword

```

1 import _ "net" // unused
2 func main() {
3     ch := make(chan string)
4     send(ch) // Oops
5     print(<-ch)
6     close(ch)
7 }
8
9 func send(ch chan string) {
10     ch <- "Hello Kent"
11 }

```

- Only one (main) goroutine
- Send without receive - blocks

Output:

```
$ go run deadlock2.go
```

Hangs: Deadlock **NOT** detected



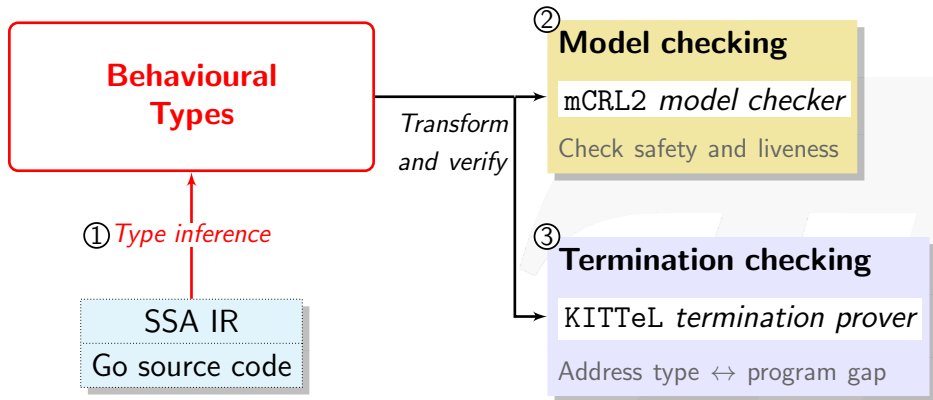
# Our goal

Check liveness/safety properties **in addition to** global deadlocks

- Apply process calculi techniques to Go
- Use model checking to statically analyse Go programs

# Behavioural type inference

## Abstract Go communication as Behavioural Types



# Infer Behavioural Types from Go program

## Go source code

```

1 func main() {
2     ch := make(chan int)
3     go send(ch)
4     print(<-ch)
5     close(ch)
6 }
7
8 func send(c chan int) {
9     c <- 1
10 }

```

## Behavioural Types

Types of CCS-like [Milner '80]  
process calculus

- Send/Receive
- new (channel)
- parallel composition (spawn)

*Go-specific*

- Close channel
- Select (guarded choice)

# Infer Behavioural Types from Go program

## Go source code

```

1 func main() {
2     ch := make(chan int)
3     go send(ch)
4     print(<-ch)
5     close(ch)
6 }
7
8 func send(c chan int) {
9     c <- 1
10 }

```

## Inferred Behavioural Types

→

$$\left\{ \begin{array}{l} \text{main()} = (\text{new } ch); \\ \quad (\text{send}\langle ch \rangle \mid \\ \quad \quad ch; \\ \quad \quad \text{close } ch), \\ \text{send}(ch) = \overline{ch} \end{array} \right\}$$

# Infer Behavioural Types from Go program

## Go source code

```

1 func main() {
2   ch := make(chan int)
3   go send(ch)
4   print(<-ch)
5   close(ch)
6 }
7
8 func send(c chan int) {
9   c <- 1
10 }
    
```

## Inferred Behavioural Types

create channel →  $\text{main}() = (\text{new } ch);$   
 spawn →  $(\text{send}\langle ch \rangle \mid$   
 receive →  $ch;$   
 close →  $\text{close } ch),$   
 $\text{send}(ch) = \overline{ch}$   
 send →

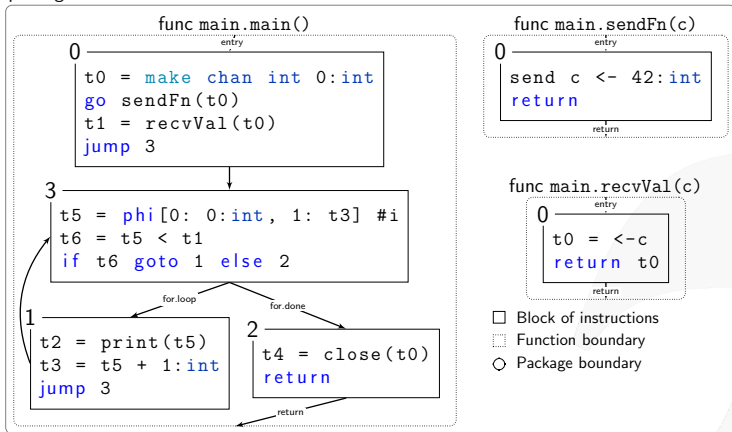


# Infer Behavioural Types from Go program

```
1 func main() {
2     ch := make(chan int) // Create channel
3     go sendFn(ch)        // Run as goroutine
4     x := recvVal(ch)     // Function call
5     for i := 0; i < x; i++ {
6         print(i)
7     }
8     close(ch) // Close channel
9 }
10 func sendFn(c chan int) { c <- 3 } // Send to c
11 func recvVal(c chan int) int { return <-c } // Recv from c
```

# Infer Behavioural Types from Go program

package main



Analyse in

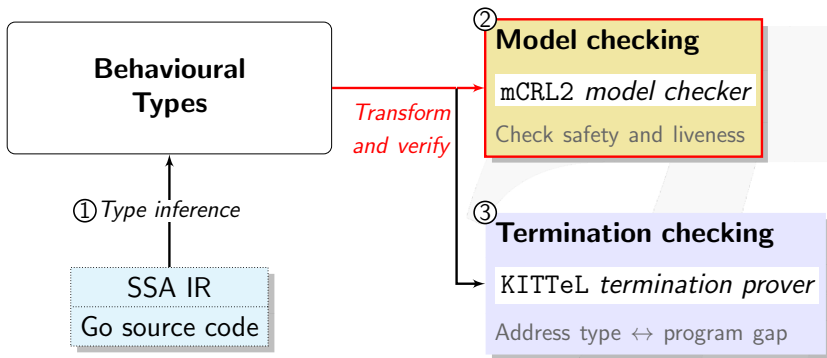
Static **S**ingle  
**A**ssignment

SSA representation  
of input program

- Only inspect **communication** primitives
- Distinguish between unique channels

# Model checking behavioural types

From behavioural types to **model** and **property specification**



# Model checking behavioural types

$$M \models \phi$$

- **LTS model** : inferred type + type semantics
  - **Safety/liveness properties** :  $\mu$ -calculus formulae for LTS
  - Check with mCRL2 model checker
    - mCRL2 constraint: *Finite control* (no spawning in loops)
- Global deadlock freedom
  - Channel safety (no send/`close` on closed channel)
  - Liveness (partial deadlock freedom)
  - Eventual reception

# Behavioural Types as LTS model

Standard CS semantics, i.e.

$$\bar{a}; T \xrightarrow{\bar{a}} T$$

Send on channel  $a$

$$\frac{T \xrightarrow{\bar{a}} T' \quad S \xrightarrow{a} S'}{T \mid S \xrightarrow{\tau_a} T' \mid S'}$$

Synchronise on  $a$

$$a; T \xrightarrow{a} T$$

Receive on channel  $a$

# Behavioural Types as LTS model

Standard CS semantics, i.e.

$$\begin{array}{c}
 \bar{a}; T \xrightarrow{\bar{a}} T \\
 \text{Send on channel } a
 \end{array}
 \quad
 \frac{T \xrightarrow{\bar{a}} T' \quad S \xrightarrow{a} S'}{T \mid S \xrightarrow{\tau_a} T' \mid S'}
 \quad
 \begin{array}{c}
 a; T \xrightarrow{a} T \\
 \text{Receive on channel } a
 \end{array}$$

Synchronise on  $a$

# Specifying **properties** of model

**Barbs** (predicates at each state) describe property at state

- Concept from process calculi [Milner '88, Sangiorgi '92]
- $\mu$ -calculus **properties** specified in terms of barbs

## Barbs ( $T \downarrow_o$ )

- Predicates of state/type  $T$
- Holds when  $T$  is ready to fire action  $o$

# Specifying properties of model

$$\begin{array}{ccc}
 \bar{a}; T \downarrow_{\bar{a}} & \frac{T \downarrow_{\bar{a}} \quad T' \downarrow_a}{T \mid T' \downarrow_{\tau_a}} & a; T \downarrow_a \\
 \text{Ready to send} & \text{Ready to synchronise} & \text{Ready to receive}
 \end{array}$$

## Barbs ( $T \downarrow_o$ )

- Predicates of state/type  $T$
- Holds when  $T$  is ready to fire action  $o$



# Specifying properties of model

$$\begin{array}{ccc}
 \bar{a}; T \downarrow_{\bar{a}} & \frac{T \downarrow_{\bar{a}} \quad T' \downarrow_a}{T \mid T' \downarrow_{\tau_a}} & a; T \downarrow_a \\
 \text{Ready to send} & \text{Ready to synchronise} & \text{Ready to receive}
 \end{array}$$

## Barbs ( $T \downarrow_o$ )

- Predicates of state/type  $T$
- Holds when  $T$  is ready to fire action  $o$

Mi Go Liveness / Safety

$P \Downarrow a$

Barb  
[Milner 8  
Sangiorgi 92]

## Channel Safety

- ▶ Channel is closed at most once
- ▶ Can only input from a closed channel (default value)
- ▶ Others raise an error and **crash**

MiGo Liveness / Safety

$P \Downarrow a$

Barb  
[Milner 8  
Sangiorgi 92]

## Channel Safety

- ▶ Channel is closed at most once
- ▶ Can only input from a closed channel (default value)
- ▶ Others raise an error and **crash**

$P$  is channel safe if  $P \xrightarrow{*} (\nu \bar{c}) Q$  and  $Q \Downarrow \text{close}(a)$

$$\neg(Q \Downarrow \text{end}(a)) \wedge \neg(Q \Downarrow \bar{a})$$

never closing

never send

a closed

# Migo Liveness / Safety

## ► Liveness

All reachable actions are eventually performed

$P$  is live if  $P \xrightarrow{*}_{(vc)} Q$

$$Q \downarrow a \Rightarrow Q \downarrow \tau \text{ at } a$$

$$Q \downarrow \bar{a} \Rightarrow Q \downarrow \tau \text{ at } a$$

Reduction  
( $\tau$ )  
at  $a$

# Select



Time  
Out

$P_1 = \text{select } \{a!, b?, z.P\}$

$P_2 = \text{select } \{a!, b?\}$

$R_1 = a?$

# Select



$P_1 = \text{select } \{a!, b?, z.P\}$

Time  
Out

if P is live  
 $P_1$  is live

$P_2 = \text{select } \{a!, b?\}$

$R_1 = a?$

# Select



$P_1 = \text{select } \{a!, b?, z.P\}$

Time  
out

if  $P$  is live  
 $P_1$  is live

$P_2 = \text{select } \{a!, b?\}$

$R_1 = a?$

$P_2$  is not  
live  
 $P_2 | R_2$  is

# Select



$P_1 = \text{select } \{a!, b?, z.P\}$

Time  
out

if  $P$  is live  
 $P_1$  is live

$P_2 = \text{select } \{a!, b?\}$

$R_1 = a?$

$P_2$  is not  
live  
 $P_2 | R_2$  is

Barb  $\downarrow \tilde{a}$

$\frac{\pi_i \downarrow a_i}{\text{select } \{\pi_i. P_i\} \downarrow \tilde{a}}$

$\frac{P \downarrow \tilde{a} \quad Q \downarrow \bar{a}_i}{P | Q \downarrow [a_i]}$

Liveness  $Q \downarrow \tilde{a} \Rightarrow Q \Downarrow z$  at  $a_i$



# Specifying **properties** of model

Given

- **LTS model** from inferred behavioural types
- **Barbs** of the LTS model

Express **safety/liveness properties**

- As  $\mu$ -calculus formulae
  - In terms of the **model** and the **barbs**
- Global deadlock freedom
  - Channel safety (no send/`close` on closed channel)
  - Liveness (partial deadlock freedom)
  - Eventual reception

# Property: Global deadlock freedom

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_a \vee \downarrow_{\bar{a}} \right) \implies \langle \mathbb{A} \rangle \text{true}$$

If a channel  $a$  is ready to **receive** or **send**,  
then there must be a **next state** (i.e. not stuck)

$\mathcal{A}$  = set of all initialised channels       $\mathbb{A}$  = set of all labels  
 $\implies$  Ready **receive/send** = not end of program.

# Property: Global deadlock freedom

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_a \vee \downarrow_{\bar{a}} \right) \implies \langle \mathbb{A} \rangle \text{true}$$

```

1  import _ "net" // unused
2  func main() {
3      ch := make(chan string)
4      send(ch) // Oops
5      print(<-ch)
6      close(ch)
7  }
8
9  func send(ch chan string) {
10     ch <- "Hello Kent"
11 }

```

- Send ( $\downarrow_{\bar{ch}}$ : line 10)
- No synchronisation
- No more reduction

# Property: Channel safety

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow a^* \right) \implies \neg(\downarrow \bar{a} \vee \downarrow \text{clo } a)$$

Once a channel  $a$  is closed ( $a^*$ ),  
it will not be **sent to**, nor closed again (**clo**  $a$ )

# Property: Channel safety

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow a^* \right) \implies \neg(\downarrow \bar{a} \vee \downarrow \text{close } a)$$

```

1 func main() {
2     ch := make(chan int)
3     go func(ch chan int) {
4         ch <- 1 // is ch closed?
5     }(ch)
6     close(ch)
7     <-ch
8 }

```

- $\downarrow \text{close } ch$  when `close(ch)`
- $\downarrow ch^*$  fires after closed
- Send ( $\downarrow \bar{ch}$ : line 4)

# Property: Liveness (partial deadlock freedom)

Liveness for Send/Receive

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_a \vee \downarrow_{\bar{a}} \right) \implies \text{eventually } (\langle \tau_a \rangle \text{true})$$

If a channel is ready to **receive** or **send**,

then **eventually**

it can synchronise  $(\tau_a)$

(i.e. there's corresponding send for **receiver**/recv for **sender**)

# Property: Liveness (partial deadlock freedom)

Liveness for Send/Receive

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_a \vee \downarrow_{\bar{a}} \right) \implies \text{eventually} (\langle \tau_a \rangle \text{true})$$

where:

$$\text{eventually} (\phi) \stackrel{\text{def}}{=} \mu \mathbf{y}. (\phi \vee \langle \mathbb{A} \rangle \mathbf{y})$$

If a channel is ready to **receive** or **send**,  
then **for some reachable state**  
it can synchronise  $(\tau_a)$

# Property: Liveness (partial deadlock freedom)

Liveness for Select

$$\left( \bigwedge_{\tilde{a} \in \mathcal{P}(\mathcal{A})} \downarrow_{\tilde{a}} \right) \implies \text{eventually} \left( \langle \{ \tau_a \mid a \in \tilde{a} \} \rangle \text{true} \right)$$

If one of the channels in `select` is ready to `receive` or `send`,  
Then **eventually** it will synchronise ( $\tau_a$ )



# Property: Liveness (partial deadlock freedom)

Liveness for Select

$$\left( \bigwedge_{\tilde{a} \in \mathcal{P}(\mathcal{A})} \downarrow_{\tilde{a}} \right) \implies \text{eventually } (\langle \{ \tau_a \mid a \in \tilde{a} \} \rangle \text{true})$$

$$P_1 = \text{select}\{\bar{a}, b, \tau.P\}$$

$$P_2 = \text{select}\{\bar{a}, b\}$$

$$R_1 = a$$

$P_1$  is live if  $P$  is ✓

$P_2$  is not live ✗

$(P_2 \mid R_1)$  is live ✓

# Property: Liveness (partial deadlock freedom)

Liveness for Select

$$\left( \bigwedge_{\tilde{a} \in \mathcal{P}(\mathcal{A})} \downarrow_{\tilde{a}} \right) \implies \text{eventually} \left( \langle \{ \tau_a \mid a \in \tilde{a} \} \rangle \text{true} \right)$$

$$P_1 = \text{select} \{ \bar{a}, b, \tau.P \}$$

$$P_2 = \text{select} \{ \bar{a}, b \}$$

$$R_1 = a$$

$P_1$  is live if  $P$  is ✓

$P_2$  is not live ✗

$(P_2 \mid R_1)$  is live ✓

# Property: Liveness (partial deadlock freedom)

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_a \vee \downarrow_{\bar{a}} \right) \implies \text{eventually} (\langle \tau_a \rangle \text{true})$$

$$\left( \bigwedge_{\tilde{a} \in \mathcal{P}(\mathcal{A})} \downarrow_{\tilde{a}} \right) \implies \text{eventually} (\langle \{ \tau_a \mid a \in \tilde{a} \} \rangle \text{true})$$

```

1 func main() {
2     ch := make(chan int)
3     go looper() // !!!
4     <-ch        // No matching send
5 }
6 func looper() {
7     for {
8     }
9 }

```

- ✗ Runtime detector: Hangs
- ✓ Our tool: NOT live

# Property: Liveness (partial deadlock freedom)

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_a \vee \downarrow_{\bar{a}} \right) \implies \text{eventually} (\langle \tau_a \rangle \text{true})$$

$$\left( \bigwedge_{\tilde{a} \in \mathcal{P}(\mathcal{A})} \downarrow_{\tilde{a}} \right) \implies \text{eventually} (\langle \{ \tau_a \mid a \in \tilde{a} \} \rangle \text{true})$$

```

1 func main() {
2     ch := make(chan int)
3     go loopSend(ch)
4     <-ch
5 }
6 func loopSend(ch chan int) {
7     for i := 0; i < 10; i-- {
8         // Does not terminate
9     }
10    ch <- 1
11 }

```

What about this one?

- Type: **Live**
- Program: **NOT** live

Needs **additional** guarantees

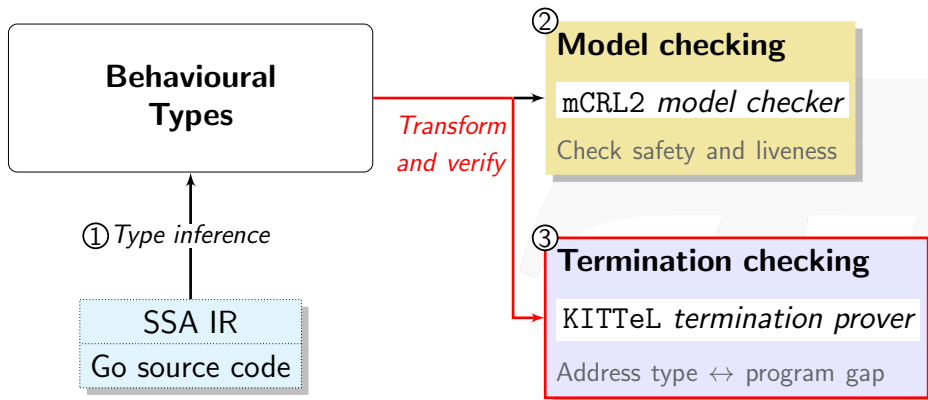
# Property: Eventual reception

$$\left( \bigwedge_{a \in \mathcal{A}} \downarrow_{a^\bullet} \right) \implies \text{eventually} (\langle \tau_a \rangle \text{true})$$

If an item is sent to a buffered channel ( $a^\bullet$ ),  
 Then **eventually** it can be consumed/synchronised ( $\tau_a$ )  
 (i.e. no orphan messages)

# Termination checking

Addressing the program-type *abstraction gap*



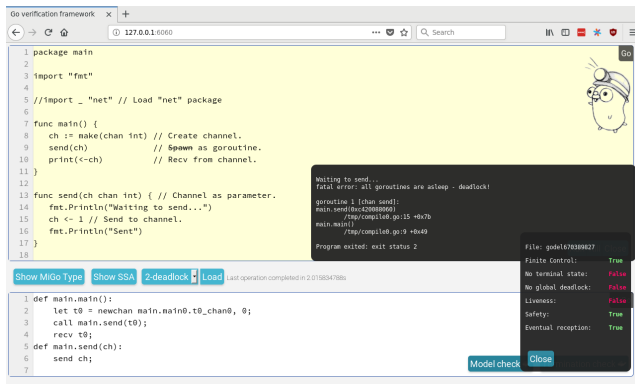
# Termination checking with KITTeL

Type inference does not consider *program data*

- Type liveness  $\neq$  Program liveness if program non-terminating
  - Especially when involving iteration
- ⇒ Check for loop termination
- If terminates, type liveness = program liveness

	Program terminates	Program does not terminate
Type live	✓ Program live	?
Type not live	✗ Program not live	✗ Program not live

# Tool: Godel-Checker



<https://github.com/nickng/gospal>

<https://bitbucket.org/MobilityReadingGroup/godel-checker>



 Understanding Concurrency with Behavioural Types

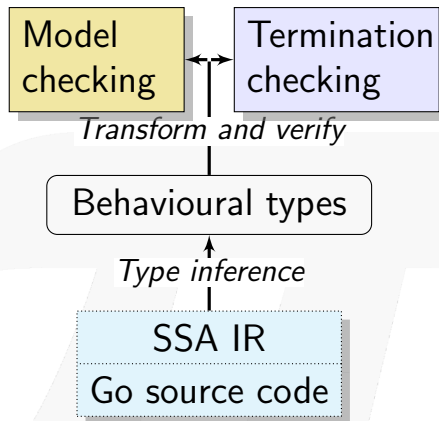
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# Conclusion

Verification framework based on  
**Behavioural Types**

- Behavioural types for Go concurrency
- Infer types from Go source code
- Model check types for safety/liveness
- + termination for iterative Go code



# In the paper

See our paper for omitted topics in this talk:

- Behavioural type inference algorithm
- Treatment of buffered (asynchronous) channels
- The `select` (non-deterministic choice) primitive
- Definitions of behavioural type semantics/barbs

Table 3: Go programs verified by our framework and comparison with existing static deadlock detection tools.

Programs	LoC	# states	Godel Checker						dingo-hunter [36]		gopherlyzer [40]		GolInfer/Gong [30]				
			$\psi_g$	$\psi_l$	$\psi_s$	$\psi_e$	Infer	Live	Live+CS	Term	Live	Time	DF	Time	Live	CS	Time
1 mismatch [36]	29	53	×	×	✓	✓	620.7	996.8	996.7	✓	×	639.4	×	3956.4	×	✓	616.8
2 fixed [36]	27	16	✓	✓	✓	✓	624.4	996.5	996.3	✓	✓	603.1	✓	3166.3	✓	✓	601.0
3 fanin [36, 39]	41	39	✓	✓	✓	✓	631.1	996.2	996.2	✓	✓	608.9	✓	19.8	✓	✓	696.7
4 sieve [30, 36]	43	∞			n/a		-	-	-	n/a	n/a	-	n/a	-	✓	✓	778.3
5 philo [40]	41	65	×	×	✓	✓	6.1	996.5	996.6	✓	×	34.2	×	27.0	×	✓	16.8
6 dinephil3 [13, 33]	55	3838	✓	✓	✓	✓	645.2	996.4	996.3	✓	n/a	-	n/a	-	✓	✓	13.2 min
7 starvephil3	47	3151	×	×	✓	✓	628.2	996.5	996.5	✓	n/a	-	n/a	-	×	✓	3.5 min
8 sel [40]	22	103	×	×	✓	✓	4.2	996.7	996.6	✓	×	15.3	×	13.0	×	✓	50.5
9 selFixed [40]	22	20	✓	✓	✓	✓	4.0	996.3	996.4	✓	✓	14.9	✓	3168.3	✓	✓	13.1
10 jobsched [30]	43	43	✓	✓	✓	✓	632.7	996.7	1996.1	✓	n/a	-	✓	4753.6	✓	✓	635.2
11 forselect [30]	42	26	✓	✓	✓	✓	623.3	996.4	996.3	✓	✓	611.8	n/a	-	✓	✓	618.6
12 cond-recur [30]	37	12	✓	✓	✓	✓	4.0	996.2	996.2	✓	✓	9.4	n/a	-	✓	✓	14.7
13 concsys [42]	118	15	×	×	✓	✓	549.7	996.5	996.4	✓	n/a	-	×	5278.6	×	✓	521.3
14 alt-bit [30, 35]	70	112	✓	✓	✓	✓	634.4	996.3	996.3	✓	n/a	-	n/a	-	✓	✓	916.8
15 prod-cons	28	106	✓	×	✓	✓	4.1	996.4	1996.2	✓	×	10.1	×	30.1	×	✓	21.8
16 nonlive	16	8	✓	✓	✓	✓	630.1	996.6	996.5	timeout	⊗	613.6	n/a	-	⊗	×	613.8
17 double-close	15	17	✓	✓	×	✓	3.5	996.6	1996.6	✓	⊗	8.7	⊗	11.8	✓	×	9.1
18 stuckmsg	8	4	✓	✓	✓	×	3.5	996.6	996.6	✓	n/a	-	n/a	-	✓	✓	7.6

# Future and related work

Extend framework to support more safety properties

Different verification approaches

- Godel-Checker model checking [ICSE'18] (this talk)
- Gong type verifier [POPL'17]
- Choreography synthesis [CC'15]

Different concurrency issues (e.g. data races)



# Distributed Programming using Role-Parametric Session Types in Go

David Castro, Raymond Hu, Sung-Shik Jongmans,  
Nicholas Ng, Nobuko Yoshida

## Scribble: Describing Multi Party Protocols

Scribble is a language to describe application-level protocols among communicating systems. A protocol represents an agreement on how participating systems interact with each other. Without a protocol, it is hard to do meaningful interaction: participants simply cannot communicate effectively, since they do not know when to expect the other parties to send data, or whether the other party is ready to receive data. However, having a description of a protocol has further benefits. It enables verification to ensure that the protocol can be implemented without resulting in unintended consequences, such as deadlocks.

### Describe

Scribble is a language for describing multiparty protocols from a global, or endpoint neutral, perspective.

### Verify

Scribble has a theoretical foundation, based on the Pi Calculus and Session Types, to ensure that protocols described using the language are sound, and do not suffer from deadlocks or livelocks.

### Project

Endpoint projection is the term used for identifying the responsibility of a particular role (or endpoint) within a protocol.

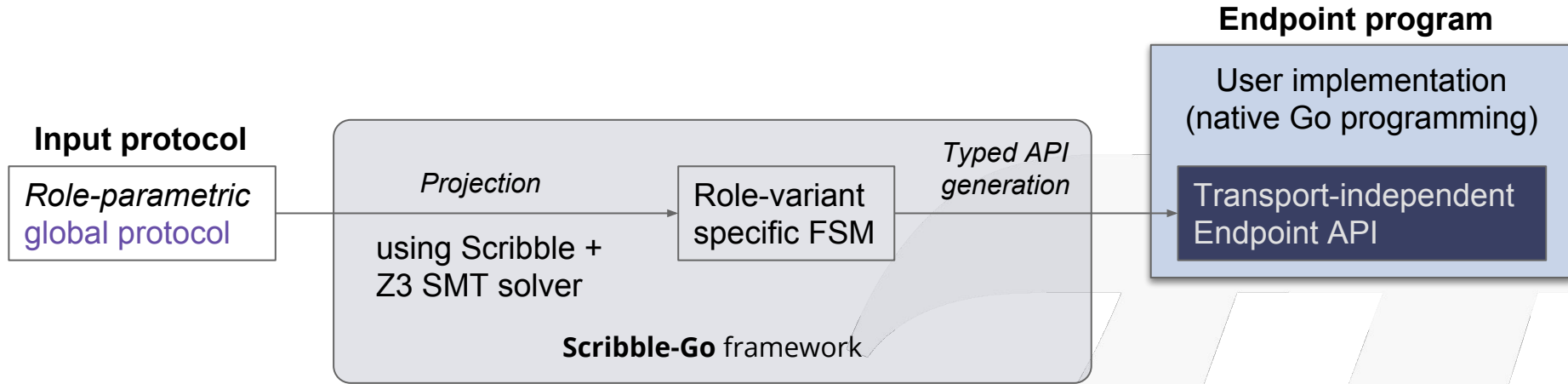
### Implement

Various options exist, including (a) using the endpoint projection for a role to generate a skeleton code, (b) using session type APIs to clearly describe the behaviour, and (c) statically verify the code against the projection.

### Monitor

Use the endpoint projection for roles defined within a Scribble protocol, to monitor the activity of a particular endpoint, to ensure it correctly implements the expected behaviour.

# Scribble-Go workflow



1. Write a *role-parametric global* protocol
2. Select endpoint *role variant* to implement (e.g. Fetcher)
3. Use **Scribble-Go** to project and generate **Endpoint API**
4. Implement endpoint (e.g. Fetcher [3]) using the **Endpoint API**

# Role variant

Role variant are *unique kinds* of endpoints

$\{M, F[1..n], \text{Server}\}$

If  $F[1]$  sends an extra request

HTTP HEAD to Server to get total size

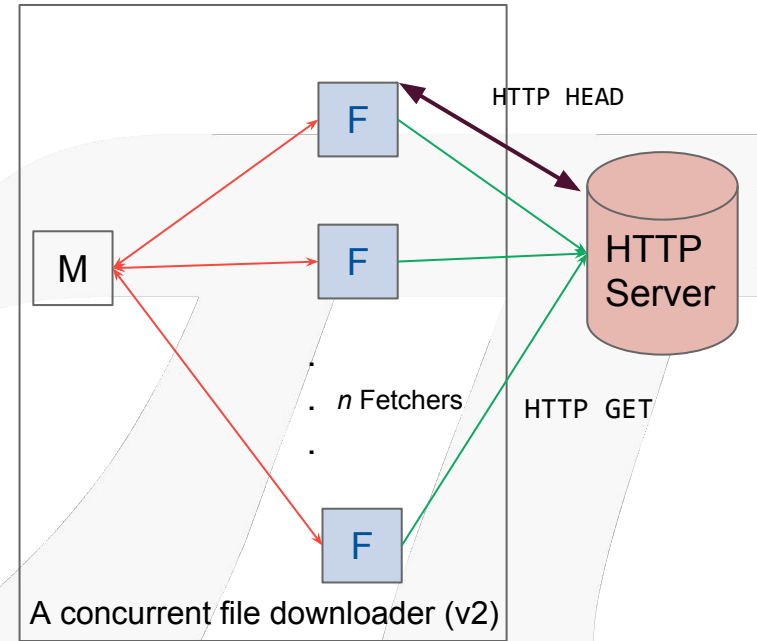
Then acts as a normal F

The role variants are:

$\{M, F[1], F[2..n], \text{Server}\}$

→  $F[1]$  and  $F[2..n]$  are different endpoints

Inference of role variants (indices): formulated as SMT constraints for Z3



# Endpoint API generation and usage

FSMs from local protocols → Message passing API

- Fluent-style
  - Every state is a unique type (struct)
  - Method calls (communication) returns next state
- Type information can be leveraged by IDEs
  - “dot-driven” content assist & auto complete

```
func doM2(m2 *M_2) M_End {  
    if m3 := m2.  
        Err: error  
        F_1_to_k: t1  
  
    func doM2(m2 *M_2) M_End {  
        if m3 := m2.F_1_to_k.  
            Scatter: t2  
  
        func doM2(m2 *M_2) M_End {  
            if m3 := m2.F_1_to_k.Scatter.  
                Job(a []Job) *M_3  
  
                :  
  
        func doM2(m2 *M_2, meta *Meta) M_End {  
            if m3 := m2.F_1_to_k.Scatter.Job(split(meta)); m3.Err != nil {  
            } else {  
                return m3.F_1_to_k.Gather.  
            }  
            Data(a []Data) M_End
```





# Behavioural Types for Go

## Type syntax

$$\begin{aligned}\alpha &:= \bar{u} \mid u \mid \tau \\ T, S &:= \alpha; T \mid T \oplus S \mid \&\{\alpha_i; T_i\}_{i \in I} \mid (T \mid S) \mid \mathbf{0} \\ &\mid (\text{new } a)T \mid \text{close } u; T \mid \mathbf{t}\langle \tilde{u} \rangle \mid [u]_k^n \mid \text{buf}[u]_{\text{closed}} \\ \mathbf{T} &:= \{\mathbf{t}(\tilde{y}_i) = T_i\}_{i \in I} \text{ in } S\end{aligned}$$

- Types of a CCS-like process calculus
- Abstracts Go concurrency primitives
  - Send/Recv, new (channel), parallel composition (spawn)
  - Go-specific: Close channel, Select (guarded choice)

# Semantics of types

$$\boxed{\text{SND}} \quad \bar{a}; T \xrightarrow{\bar{a}} T \quad \boxed{\text{RCV}} \quad a; T \xrightarrow{a} T \quad \boxed{\text{TAU}} \quad \tau; T \xrightarrow{\tau} T$$

$$\boxed{\text{END}} \quad \text{close } a; T \xrightarrow{\text{clo } a} T \quad \boxed{\text{BUF}} \quad [a]_k^n \xrightarrow{\overline{\text{clo } a}} \text{buf}[a]_{\text{closed}} \quad \boxed{\text{CLD}} \quad \text{buf}[a]_{\text{closed}} \xrightarrow{a^*} \text{buf}[a]_{\text{closed}}$$

$$\boxed{\text{SEL}} \quad \frac{i \in \{1, 2\}}{T_1 \oplus T_2 \xrightarrow{\tau} T_i} \quad \boxed{\text{BRA}} \quad \frac{\alpha_j; T_j \xrightarrow{\alpha_j} T_j \quad j \in I}{\&\{\alpha_i; T_i\}_{i \in I} \xrightarrow{\alpha_j} T_j}$$

$$\boxed{\text{PAR}} \quad \frac{T \xrightarrow{\alpha} T'}{T \mid S \xrightarrow{\alpha} T' \mid S} \quad \boxed{\text{SEQ}} \quad \frac{T \xrightarrow{\alpha} T'}{T; S \xrightarrow{\alpha} T'; S} \quad \boxed{\text{TERM}} \quad \mathbf{0}; S \xrightarrow{\tau} S$$

$$\boxed{\text{COM}} \quad \frac{\alpha \in \{\bar{a}, a^*, a^\bullet\} \quad T \xrightarrow{\alpha} T' \quad S \xrightarrow{\beta} S' \quad \beta \in \{a, a^\bullet\}}{T \mid S \xrightarrow{\tau_a} T' \mid S'}$$

$$\boxed{\text{EQ}} \quad \frac{T \equiv_{\alpha} T' \quad T \xrightarrow{\alpha} T''}{T' \xrightarrow{\alpha} T''}$$

$$\boxed{\text{DEF}} \quad \frac{T \{\tilde{a}/\tilde{x}\} \xrightarrow{\alpha} T' \quad \mathbf{t}(\tilde{x}) = T}{\mathbf{t}(\tilde{a}) \xrightarrow{\alpha} T'}$$

$$\boxed{\text{CLOSE}} \quad \frac{T \xrightarrow{\text{clo } a} T' \quad S \xrightarrow{\overline{\text{clo } a}} S'}{T \mid S \xrightarrow{\tau} T' \mid S'}$$

$$\boxed{\text{IN}} \quad \frac{k < n}{[a]_k^n \xrightarrow{a^\bullet} [a]_{k+1}^n} \quad \boxed{\text{OUT}} \quad \frac{k \geq 1}{[a]_k^n \xrightarrow{a^\bullet} [a]_{k-1}^n}$$

# Barb predicates for types

$$\begin{array}{c}
 a; T \downarrow_a \quad \text{close } a; T \downarrow_{\text{clo } a} \quad \frac{\forall i \in \{1, \dots, n\} : \alpha_i \downarrow_{o_i}}{\&\{\alpha_i; T\}_{i \in \{1, \dots, n\}} \downarrow_{\{o_1 \dots o_n\}}} \\
 \bar{a}; T \downarrow_{\bar{a}} \quad \text{buf}[a]_{\text{closed}} \downarrow_{a^*} \\
 \\
 \frac{T \downarrow_o}{T; T' \downarrow_o} \quad \frac{T \downarrow_a \quad T' \downarrow_{\bar{a}} \text{ or } T' \downarrow_{a^*}}{T \mid T' \downarrow_{\tau_a}} \quad \frac{T \{\bar{a}/\bar{x}\} \downarrow_o \quad \mathbf{t}(\bar{x}) = T}{\mathbf{t}(\bar{a}) \downarrow_o} \\
 \\
 \frac{T \downarrow_a \quad \alpha_i \downarrow_{\bar{a}}}{T \mid \&\{\alpha_i; S_i\}_{i \in I} \downarrow_{\tau_a}} \quad \frac{T \downarrow_{\bar{a}} \text{ or } T \downarrow_{a^*} \quad \alpha_i \downarrow_a}{T \mid \&\{\alpha_i; S_i\}_{i \in I} \downarrow_{\tau_a}} \\
 \\
 \frac{k < n}{[a]_k^n \downarrow_{\bullet a}} \quad \frac{k \geq 1}{[a]_k^n \downarrow_{a^\bullet}} \quad \frac{T \downarrow_{\bar{a}} \quad T' \downarrow_{\bullet a}}{T \mid T' \downarrow_{\tau_a}} \quad \frac{T \downarrow_{a^\bullet} \quad \alpha_i \downarrow_a}{T \mid \&\{\alpha_i; S_i\}_{i \in I} \downarrow_{\tau_a}} \\
 \\
 \frac{T \downarrow_o}{T \mid T' \downarrow_o} \quad \frac{T \downarrow_o \quad a \notin \text{fn}(o)}{(\text{new}^n a); T \downarrow_o} \quad \frac{T \downarrow_o \quad T \equiv T'}{T \downarrow_o}
 \end{array}$$

Figure: Barb predicates for types.