On Asynchronous Session Semantics

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Abstract. This paper studies a behavioural theory of the π-calculus with session types under the fundamental principles of the practice of distributed computing — asynchronous communication which is order-preserving inside each connection (session), augmented with asynchronous inspection of events (message arrivals). A new theory of bisimulations is introduced, distinct from either standard asynchronous or synchronous bisimilarity, accurately capturing the semantic nature of session-based asynchronously communicating processes augmented with event primitives. The bisimilarity coincides with the reduction-closed barbed congruence. We examine its properties and compare them with existing semantics. Using the behavioural theory, we verify that the program transformation of multithreaded into event-driven session based processes, using Lauer-Needham duality, is type and semantic preserving. Our benchmark results in Session-based Java demonstrate the potential of the session-type based translation as semantically transparent optimisation techniques.

1 Introduction

Modern transports such as TCP in distributed networks provide reliable, ordered delivery of messages from a program on one computer to another, once a connection is established. In practical communications programming, two parties start a conversation by establishing a connection over such a transport and exchange semantically meaningful, formatted messages through this connection. The distinction between possibly non order-preserving communications outside of connection and order-preserving ones inside each connection is a key feature of this practice: order preservation allows proper handling of a sequence of messages following an agreed-upon conversation structure, while unordered deliveries across connections enhance asynchronous, efficient bandwidth usage. Further, asynchronous event processing [17] using locally *buffered* messages, the receiver can asynchronously *inspect* and *consume* events/messages.

This paper investigates semantic foundations of asynchronously communicating processes, capturing these key elements of modern communications programming – distinction between non order-preserving communications outside connections and the order-preserving ones inside each connection, as well as the incorporation of asynchronous inspection of message arrivals. We use the π-calculus augmented with session primitives, buffers and a simple event inspection primitive. Typed sessions capture structured conversations in connections with type safety; while a buffer represents an intermediary between a process and its environment, capturing non-blocking nature of communications, and enabling asynchronous event processing. The formalism is intended to be an idealised, core but expressive communications programming language, offering a basis for a tractable semantic study. Our study shows that the combination of these basic elements for modern communications programming leads to a rich behavioural theory which differs from both the standard synchronous communications semantics and the fully asynchronous one [7], captured through novel equational laws for asynchrony. These laws can then be used as a semantic justification of a well-known program transformation based on Lauer and Needham's duality principle [14], which translates multi-threaded programs to their equivalent single-threaded, asynchronous, event-based programs. This transformation is regularly

used in practice, albeit in an ad-hoc manner, playing a key role in e.g. high-performance servers. Our translation is given formally, is type-preserving and is backed up by a rigorous semantic justification. While we do not detail in the main sections, the transform is implemented in the session-extension of Java [12, 13], resulting in highly competitive performance in comparison with more ad-hoc transformations.

Let us outline some of the key technical ideas informally. In the present theory, the asynchronous order-preserving communications over a connection are modelled as *asynchronous session communication*, extending the synchronous session calculus [8, 23] with *message queues* [4, 5, 11]. A message queue, written $s[\mathtt{i}:\vec{h},\mathtt{o}:\vec{h}']$, encapsulates input buffer ($\mathtt{i}$) with elements $\vec{h}$ and output buffer ($\mathtt{o}$) with $\vec{h}'$. Fig. 1 represents the two end points of a session. A message v is first enqueued by a sender $s!\langle v\rangle;P$ at its output queue at s, which intuitively represents a communication pipe extending from the sender's locality to the receiver's. The message will eventually reach the receiver's locality, formalised as its transfer from the sender's output buffer (at s) to the receiver's input buffer (at $\overline{s}$). For a receiver, only when this transfer takes place a visible (and asynchronous) message reception takes place, since only then the receiver can *inspect* and *consume* the message (as shown in **Remote** in Figure 1). Note that dequeuing and enqueing actions inside a location are local to each process and is therefore invisible (τ-actions) (**Local** in Figure 1).

Local (the dashed arrows)

$$s!\langle v\rangle;Q \mid s[\mathtt{o}:\vec{h}] \xrightarrow{\tau} Q \mid s[\mathtt{o}:\vec{h}\cdot v]$$

$$s?(x).P \mid s[\mathtt{i}:w\cdot\vec{h}] \xrightarrow{\tau} P\{w/x\} \mid s[\mathtt{i}:\vec{h}]$$

Remote (the solid arrows)

$$s[\mathtt{i}:\vec{h}] \xrightarrow{s?\langle v\rangle} s[\mathtt{i}:\vec{h}\cdot v] \qquad s[\mathtt{o}:v\cdot\vec{h}] \xrightarrow{s!\langle v\rangle} s[\mathtt{o}:\vec{h}]$$

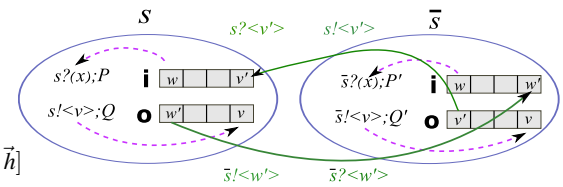

Fig. 1. The transitions in the two locations.

The induced semantics captures the nature of asynchronous observables not studied before. For example, in weak asynchronous bisimilarity ($\approx_a$ in [7, 9]), the message order is not observable ($s!\langle v_1\rangle \mid s!\langle v_2\rangle \approx_a s!\langle v_2\rangle \mid s!\langle v_1\rangle$) but in our semantics, messages for the same destination do *not* commute ($s!\langle v_1\rangle;s!\langle v_2\rangle \not\approx s!\langle v_2\rangle;s!\langle v_1\rangle$) as in the synchronous semantics [19] ($\approx_s$ in [7, 9]); whereas two inputs for different targets commute ($s_1?(x);s_2?(y);P \approx s_2?(x);s_1?(y);P$) since the dequeue action is un-observable, differing from the synchronous semantics, $s_1?(x);s_2?(y);P \not\approx_s s_2?(x);s_1?(y);P$.

Asynchronous event-handling [12] introduces further subtleties in observational laws. Asynchronous event-based programming is characterised by reactive flows driven by the detection of *events*, that is *message arrivals* at local buffers. In our formalism, this facility is distilled as a simple `arrived` predicate: for example, $Q = \mathtt{if\ arrived}\ s\ \mathtt{then}\ P_1\ \mathtt{else}\ P_2$ reduces to P_1 if the s input buffer contains one or more message; otherwise Q reduces to P_2. By `arrived`, we can observe the *movement of messages between two locations*. For example, $Q \mid s[\mathtt{i}:\emptyset] \mid \overline{s}[\mathtt{o}:v]$ is not equivalent with $Q \mid s[\mathtt{i}:v] \mid \overline{s}[\mathtt{o}:\emptyset]$ because the former can reduce to P_2 (since v has not arrived at the local buffer at s yet) while the latter cannot. To capture `arrived`, we need the IO-buffers at each session, with which one has, for example, $s_1!\langle v_1\rangle;s_2!\langle v_2\rangle \approx s_2!\langle v_2\rangle;s_1!\langle v_1\rangle$ in the presence of the `arrived` (we cannot observe the remote process performing the output), while one would have $s_1!\langle v_1\rangle;s_2!\langle v_2\rangle \not\approx s_2!\langle v_2\rangle;s_1!\langle v_1\rangle$ with the `arrived`, *if* we were without local queues.

By capturing the major elements of practical communications programming, i.e. asynchrony, ordered and unordered communications and event handling, the induced asynchronous behavioural equivalence can justify various syntactic transformations, including a widely used but hitherto unjustified program transformation for large-scale servers.

$$
\begin{array}{l}
\text{(Identifiers) } u ::= a,b \mid x,y \quad k ::= s,\overline{s} \mid x,y \quad n ::= a,b \mid s,\overline{s} \qquad \text{(Values) } v ::= \mathtt{tt},\mathtt{ff} \mid a,b \mid s,\overline{s} \\
\text{(Expressions) } \quad e ::= v \mid x,y,z \mid \mathtt{arrived}\, u \mid \mathtt{arrived}\, k \mid \mathtt{arrived}\, k\, h \\
\text{(Processes) } P,Q ::= u(x).P \mid \overline{u}(x);P \mid k!\langle e\rangle;P \mid k?(x).P \mid k \lhd l;P \mid k \rhd \{l_i : P_i\}_{i\in I} \\
\qquad\qquad \mid \quad \mathtt{if}\, e\, \mathtt{then}\, P\, \mathtt{else}\, Q \mid (\nu\, a)P \mid P \mid Q \mid \mathbf{0} \mid \mu X.P \mid X \\
\qquad\qquad \mid \quad a[\vec{s}] \mid \overline{a}\langle s\rangle \mid (\nu\, s)P \mid s[\mathtt{i}:\vec{h},\mathtt{o}:\vec{h}'] \qquad \text{(Messages) } \quad h ::= v \mid l
\end{array}
$$

Fig. 2. The syntax of processes.

Contributions Section 3 defines a bisimulation for the asynchronous eventful session calculus, examines its properties against the standard bisimulations and proves that it coincides with the barbed reduction-based congruence [9]. Section 4 provides the semantics-preserving Lauer-Needham transformation of multithreaded into event-driven processes and proves its correctness. The paper concludes with the related work with the detailed comparisons with the existing semantics. Appendix lists the full definition of Lauer-Needham transformation, the detailed definitions and full proofs. Appendix H also gives the performance results with extensive benchmarks for justifying the transformation in Session-based Java implementation [12]. Appendix and [22] are provided only for the reviewers' convenience: the paper is self-contained and can be read without them.

2 Asynchronous Network Communications in Sessions

2.1 Syntax

We use a sub-calculus of the eventful session π-calculus [12]. In Figure 2, values $v, v', ...$ include constants, *shared channels* a, b, c, and *session channels* s, s'. A session channel denotes one endpoint of a session: s and $\overline{s}$ denote two ends of a single session, with $\overline{\overline{s}} = s$. Labels for branching and selection range over $l, l', ...$, variables over x, y, z, and process variables over X, Y, Z. Shared channel identifiers u, u' denote shared channels/variables; session identifiers k, k' are session endpoints and variables. n denotes either a or s. Expressions e are values, variables and the message arrival predicates ($\mathtt{arrived}\, u$, $\mathtt{arrived}\, k$ and $\mathtt{arrived}\, k\, h$: the last one checks for the arrival of the specific message h at k). $\vec{s}$ and $\vec{h}$ stand for vectors of session channels and messages respectively. ε denotes the empty vector.

We distinguish two kinds of asynchronous communications, *asynchronous session initiation* and *asynchronous session communication* (over an established session). The former involves the *unordered* delivery of a *session request message* $\overline{a}\langle s\rangle$, where $\overline{a}\langle s\rangle$ represents an asynchronous message in transit towards an acceptor at a, carrying a fresh session channel s. As in actual network, a request message will first move through the network and eventually get buffered at a receiver's end. Only then a message arrival can be detected. This aspect is formalised by the introduction of a *shared channel input queue* $a[\vec{s}]$, often called *shared input queue* for brevity, which denotes an acceptor's local buffer at a with pending session requests for $\vec{s}$. The intuitive meaning of the end-point configuration $s[\mathtt{i}:\vec{h},\mathtt{o}:\vec{h}']$ is explained in Introduction.

Requester $\overline{u}(x);P$ requests a session initiation, while acceptor $u(x).P$ accepts one. Through an established session, output $k!\langle e\rangle;P$ sends e through channel k asynchronously, input $k?(x).P$ receives through k, selection $k \lhd l;P$ chooses the branch with label l, and branching $k \rhd \{l_i : P_i\}_{i\in I}$ offers branches. The $(\nu\, a)P$ binds a channel a, while $(\nu\, s)P$ binds the two endpoints, s and $\overline{s}$, making them private within P. The conditional, parallel composition, recursions and inaction are standard. $\mathbf{0}$ is often omitted. For brevity, one or more components may be omitted from a configuration when they are irrelevant, writing e.g. $s[\mathtt{i}:\vec{h}]$

$$
\begin{array}{ll}
\text{[Request1]} & \overline{a}(x);P \longrightarrow (\nu\, s)\,(P\{\overline{s}/x\} \mid \overline{s}\,[\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid \overline{a}\langle s\rangle) \quad (s \notin \mathrm{fn}(P)) \\
\text{[Request2]} & a\,[\vec{s}] \mid \overline{a}\,\langle s\rangle \longrightarrow a\,[\vec{s}\cdot s] \\
\text{[Accept]} & a(x).P \mid a\,[s\cdot\vec{s}] \longrightarrow P\{s/x\} \mid s\,[\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid a\,[\vec{s}] \\
\text{[Send,Recv]} & s!\langle v\rangle;P \mid s\,[\mathtt{o}:\vec{h}] \longrightarrow P \mid s\,[\mathtt{o}:\vec{h}\cdot v] \qquad s?(x).P \mid s\,[\mathtt{i}:v\cdot\vec{h}] \longrightarrow P\{v/x\} \mid s\,[\mathtt{i}:\vec{h}] \\
\text{[Sel,Bra]} & s \lhd l_i;P \mid s\,[\mathtt{o}:\vec{h}] \longrightarrow P \mid s\,[\mathtt{o}:\vec{h}\cdot l_i] \quad s \rhd \{l_j:P_j\}_{j\in J} \mid s\,[\mathtt{i}:l_i\cdot\vec{h}] \longrightarrow P_i \mid s\,[\mathtt{i}:\vec{h}] \quad (i\in J) \\
\text{[Comm]} & s\,[\mathtt{o}:v\cdot\vec{h}] \mid \overline{s}\,[\mathtt{i}:\vec{h}'] \longrightarrow s\,[\mathtt{o}:\vec{h}] \mid \overline{s}\,[\mathtt{i}:\vec{h}'\cdot v] \\
\text{[Areq]} & E[\mathtt{arrived}\ a] \mid a\,[\vec{s}] \longrightarrow E[\mathtt{b}] \mid a\,[\vec{s}] \qquad (|\vec{s}| \geq 1) \searrow \mathtt{b} \\
\text{[Ases]} & E[\mathtt{arrived}\ s] \mid s\,[\mathtt{i}:\vec{h}] \longrightarrow E[\mathtt{b}] \mid s\,[\mathtt{i}:\vec{h}] \qquad (|\vec{h}| \geq 1) \searrow \mathtt{b} \\
\text{[Amsg]} & E[\mathtt{arrived}\ s\ h] \mid s\,[\mathtt{i}:\vec{h}] \longrightarrow E[\mathtt{b}] \mid s\,[\mathtt{i}:\vec{h}] \qquad (\vec{h} = h\cdot\vec{h}') \searrow \mathtt{b}
\end{array}
$$

Fig. 3. Selected reduction rules.

which denotes the input part of $s\,[\mathtt{i}:\vec{h},\mathtt{o}:\vec{h}']$. The notions of free variables and channels are standard [20]; we write $\mathrm{fn}(P)$ for the set of free channels in P. The syntax $\overline{a}\langle s\rangle$, $(\nu\, s)\,P$ and $s\,[\mathtt{i}:\vec{h},\mathtt{o}:\vec{h}']$ only appear at runtime. A process without free variables is called *closed* and a closed process without runtime syntax is called *program*.

2.2 Operational Semantics

Reduction is defined over closed terms. The key rules are given in Figure 3. We use the standard evaluation contexts $E[\_]$ defined as $E ::= \ -\ \mid\ s!\langle E\rangle;P\ \mid\ \mathtt{if}\ E\ \mathtt{then}\ P\ \mathtt{else}\ Q$. The structural congruence $\equiv$ and the rest of the reduction rules are standard.

The first three rules define the initialisation. In [Request1], a client requests a server for a fresh session via shared channel a. A fresh session channel, with two ends s (server-side) and $\overline{s}$ (client-side) as well as the empty configuration at the client side, are generated and the session request message $\overline{a}\langle s\rangle$ is dispatched. Rule [Request2] enqueues the request in the shared input queue at a. A server accepts a session request from the queue using [Accept], instantiating its variable with s in the request message; the new session is now established.

Asynchronous order-preserving session communications are modelled by the next four rules. Rule [Send] enqueues a value in the o-buffer at the *local* configuration; rule [Receive] dequeues the first value from the i-buffer at the local configuration; rules [Sel] and [Bra] similarly enqueue and dequeue a label. The arrival of a message at a remote site is embodied by [Comm], which removes the first message from the o-buffer of the sender configuration and enqueues it in the i-buffer at the receiving configuration. Note the first four rules manipulate only the local configurations.

Output actions are always non-blocking. An input action can block if no message is available at the corresponding local input buffer. The use of the message arrivals can avoid this blocking: [Areq] evaluates $\mathtt{arrived}\ a$ to $\mathtt{tt}$ iff the queue is non-empty; similarly for $\mathtt{arrived}\ k$ in [Areq]. [Amsg] evaluates $\mathtt{arrived}\ s\ h$ to $\mathtt{tt}$ iff the buffer is nonempty and its next message matches h. The notation $e \searrow \mathtt{b}$ means e evaluates to $\mathtt{b}$; and $\twoheadrightarrow = (\longrightarrow \cup \equiv)^*$.

2.3 Types and Typing

The type syntax follows the standard session types from [8].

$$
\begin{array}{lll}
\text{(Shared)} & U ::= \mathsf{bool} \mid \mathtt{i}\langle S\rangle \mid \mathtt{o}\langle S\rangle \mid \mathsf{X} \mid \mu\mathsf{X}.U & \text{(Value)} \quad T ::= U \mid S \\
\text{(Session)} & S ::= !(T);S \mid ?(T);S \mid \oplus\{l_i:S_i\}_{i\in I} \mid \&\{l_i:S_i\}_{i\in I} \mid \mu\mathsf{X}.S \mid \mathsf{X} \mid \mathtt{end} &
\end{array}
$$

The shared types U include booleans bool (and, in examples, naturals nat); shared channel types $\mathtt{i}\langle S\rangle$ (input) and $\mathtt{o}\langle S\rangle$ (output) for shared channels through which a session of type S is established; type variables $(\mathsf{X},\mathsf{Y},\mathsf{Z},..)$; and recursive types. The IO-types (often called

server/client types) ensure a unique server and many clients [10]. In the present work they are used for controlling locality (queues are placed only at the server sides) and associated typed transitions, playing a central role in our behavioural theory. In session types, output type $!(T);S$ represents outputting values of type T, then performing as S. Dually for input type $?(T);S$. Selection type $\oplus\{l_i : S_i\}_{i\in I}$ describes a selection of one of the labels say l_i then behaves as T_i. Branching type $\&\{l_i : S_i\}_{i\in I}$ waits with I options, and behaves as type T_i if i-th label is chosen. End type $\mathtt{end}$ represents the session completion and is often omitted. In recursive type $\mu X.S$, type variables are guarded in the standard sense.

The judgements of processes and expressions are defined as:

$$\Gamma \vdash P \triangleright \Delta \text{ and } \Gamma, \Delta \vdash e : T \text{ with } \Gamma ::= \emptyset \mid \Gamma \cdot u : U \mid \Gamma \cdot X : \Delta \text{ and } \Delta ::= \emptyset \mid \Delta \cdot a \mid \Delta \cdot k : \mathsf{T} \mid \Delta \cdot s$$

where session type is extended to $\mathsf{T} ::= M;S \mid M$ with $M ::= \emptyset \mid \oplus l \mid \&l \mid !(T) \mid ?(T) \mid M;M$ which represents types for values stored in queues (note $\emptyset;S = S$). Γ is called *shared environment*, which maps shared channels and process variables to, respectively, constant types and value types; Δ is called *linear environment* maps session channels to session types and recording shared channels for acceptor's input queues and session channels for end-point queues. The judgement is read: program P is typed under shared environment Γ, uses channels as linear environment Δ. In the expression judgement, expression e has type T under Γ, and uses channels as linear environment Δ. We often omit Δ if it is clear from the context. The typing system is similar with [2, 12], thus we leave them to Appendix B. We say that Δ *well configured* if $s : S \in \Delta$, then $\overline{s} : \overline{S} \in \Delta$. We define: $\{s : !(T);S \cdot \overline{s} : ?(T);S'\} \longrightarrow \{s : S \cdot \overline{s} : S'\}$, $\{s : \oplus\{l_i : S_i\}_{i\in I} \cdot \overline{s} : \&\{l_i : S'_i\}_{i\in I}\} \longrightarrow \{s : S_k \cdot \overline{s} : S'_k\}$ $(k \in I)$, and $\Delta \cup \Delta'' \longrightarrow \Delta' \cup \Delta''$ if $\Delta \longrightarrow \Delta'$. We set $\twoheadrightarrow = \longrightarrow^*$.

Proposition 2.1 (Subject Reduction). *if $\Gamma \vdash P \triangleright \Delta$ and $P \twoheadrightarrow Q$ and Δ is well-configured, then we have $\Gamma \vdash P \triangleright \Delta'$ such that $\Delta \twoheadrightarrow \Delta'$ and Δ' is well-configured.*

In the following we study semantic properties of typed processes: however these developments can be understood without knowing the details of the typing rules.[1]

3 Asynchronous Session Bisimulations and its Properties

3.1 Labelled Transitions and Bisimilarity

$$\langle \mathsf{Acc} \rangle \quad a[\vec{s}] \xrightarrow{a\langle s \rangle} a[\vec{s} \cdot s] \qquad \langle \mathsf{Req} \rangle \quad \overline{a}\langle s \rangle \xrightarrow{\overline{a}\langle s \rangle} \mathbf{0} \qquad \langle \mathsf{In} \rangle \quad s[\mathtt{i} : \vec{h}] \xrightarrow{s?\langle v \rangle} s[\mathtt{i} : \vec{h} \cdot v]$$

$$\langle \mathsf{Out} \rangle \quad s[\mathtt{o} : v \cdot \vec{h}] \xrightarrow{s!\langle v \rangle} s[\mathtt{o} : \vec{h}] \qquad \langle \mathsf{Bra} \rangle \quad s[\mathtt{i} : \vec{h}] \xrightarrow{s\&l} s[\mathtt{i} : \vec{h} \cdot l] \qquad \langle \mathsf{Sel} \rangle \quad s[\mathtt{o} : l \cdot \vec{h}] \xrightarrow{s\oplus l} s[\mathtt{o} : h]$$

$$\langle \mathsf{Local} \rangle \frac{P \longrightarrow Q}{P \xrightarrow{\tau} Q} \quad \langle \mathsf{Par} \rangle \frac{P \xrightarrow{\ell} P' \ \mathsf{bn}(\ell) \cap \mathsf{fn}(Q) = \emptyset}{P|Q \xrightarrow{\ell} P'|Q} \quad \langle \mathsf{Tau} \rangle \frac{P \xrightarrow{\ell} P' \ Q \xrightarrow{\ell'} Q' \ \ell \asymp \ell'}{P|Q \xrightarrow{\tau} (\nu\, \mathsf{bn}(\ell, \ell'))(P'|Q')}$$

$$\langle \mathsf{Res} \rangle \frac{P \xrightarrow{\ell} P' \ n \notin \mathsf{fn}(\ell)}{(\nu\, n)P \xrightarrow{\ell} (\nu\, n)P'} \quad \langle \mathsf{OpenS} \rangle \frac{P \xrightarrow{\overline{a}\langle s \rangle} P'}{(\nu\, s)P \xrightarrow{\overline{a}\langle s \rangle} P'} \quad \langle \mathsf{OpenN} \rangle \frac{P \xrightarrow{s!\langle a \rangle} P'}{(\nu\, a)P \xrightarrow{s!\langle a \rangle} P'} \quad \langle \mathsf{Alpha} \rangle \frac{P \equiv_\alpha P' \ P' \xrightarrow{\ell} Q}{P \xrightarrow{\ell} Q}$$

Fig. 4. Labelled transition system (we omit the symmetric rule of Par).

[1] This is because the properties of the typing system are captured by the typed LTS defined in Figure 5 later.

Untyped and Typed LTS. This section studies the basic properties of behavioural equivalences. We use the following labels $(\ell, \ell', ...)$:

$$\ell \ ::= \ a\langle s\rangle \mid \overline{a}\langle s\rangle \mid \overline{a}(s) \mid s?\langle v\rangle \mid s!\langle v\rangle \mid s!(a) \mid s\&l \mid s\oplus l \mid \tau$$

where the labels denote the session accept, request and bound request, input, output, bound output, branching, selection and the τ-action. $\mathsf{sbj}(\ell)$ denotes the set of free subjects in ℓ; and $\mathsf{fn}(\ell)$ (resp. $\mathsf{bn}(\ell)$) denotes the set of free (resp. bound) names in ℓ. The symmetric operator $\ell \asymp \ell'$ on labels that denotes that ℓ is a dual of ℓ', is defined as: $a\langle s\rangle \asymp \overline{a}\langle s\rangle$, $a\langle s\rangle \asymp \overline{a}(s)$, $s?\langle v\rangle \asymp \overline{s}!\langle v\rangle$, $s?\langle a\rangle \asymp \overline{s}!(a)$, and $s\&l \asymp \overline{s}\oplus l$.

Figure 4 gives the untyped labelled transition system (LTS). $\langle\mathsf{Acc}\rangle/\langle\mathsf{Req}\rangle$ are for the session initialisation. The next four rules $\langle\mathsf{In}\rangle/\langle\mathsf{Out}\rangle/\langle\mathsf{Bra}\rangle/\langle\mathsf{Sel}\rangle$ say the action is observable when it moves from its local queue to its remote queue. When the process accesses its local queue, the action is *invisible* from the outside, as formalised by $\langle\mathsf{Local}\rangle$. Other compositional rules are standard. Based on the LTS, we use the standard notations [18] such as $P \stackrel{\ell}{\Longrightarrow} Q$, $P \stackrel{\vec{\ell}}{\Longrightarrow} Q$ and $P \stackrel{\hat{\ell}}{\Longrightarrow} Q$.

We define the typed LTS on the basis of the untyped one. The basic idea is *to use the type information to control the enabling of actions*. This is realised by introducing the definition of the *environment transition*, defined in Figure 5. A transition $(\Gamma,\Delta) \stackrel{\ell}{\longrightarrow} (\Gamma',\Delta')$ means that an environment (Γ,Δ) allows an action ℓ to take place, and the resulting environment is (Γ',Δ'), constraining process transitions through the linear and shared environments. This constraint is at the heart of our typed LTS, accurately capturing interactions in the presence of sessions and local buffers. We write $\Gamma_1 \vdash P_1 \triangleright \Delta_1 \stackrel{\ell}{\longrightarrow} \Gamma_2 \vdash P_2 \triangleright \Delta_2$ if $P_1 \stackrel{\ell}{\longrightarrow} P_2$ and $(\Gamma_1,\Delta_1) \stackrel{\ell}{\longrightarrow} (\Gamma_2,\Delta_2)$ with $\Gamma_i \vdash P_i \triangleright \Delta_i$. Similarly for other transition relations.

$$\begin{array}{rl}
\Gamma(a) = \mathtt{i}\langle S\rangle, a \in \Delta, s \text{ fresh implies} & (\Gamma,\Delta) \stackrel{a\langle s\rangle}{\longrightarrow} (\Gamma, \Delta\cdot s:\overline{S}) \\
\Gamma(a) = \mathtt{o}\langle S\rangle, a \notin \Delta \text{ implies} & (\Gamma,\Delta) \stackrel{\overline{a}\langle s\rangle}{\longrightarrow} (\Gamma,\Delta) \\
\Gamma(a) = \mathtt{o}\langle S\rangle, a \notin \Delta, s \text{ fresh implies} & (\Gamma,\Delta) \stackrel{\overline{a}(s)}{\longrightarrow} (\Gamma, \Delta\cdot s:S) \\
\Gamma \vdash v:U \text{ and } U \neq \mathtt{i}\langle S'\rangle \text{ and } \overline{s} \notin \mathrm{dom}(\Delta) \text{ implies} & (\Gamma,\Delta\cdot s:!(U);S) \stackrel{s!\langle v\rangle}{\longrightarrow} (\Gamma,\Delta\cdot s:S) \\
\overline{s} \notin \mathrm{dom}(\Delta) \text{ implies} & (\Gamma,\Delta\cdot s:!(\mathtt{o}\langle S'\rangle);S) \stackrel{s!(a)}{\longrightarrow} (\Gamma\cdot a:\mathtt{o}\langle S'\rangle,\Delta\cdot s:S) \\
\Gamma \vdash v:U \text{ and } U \neq \mathtt{i}\langle S'\rangle \text{ and } \overline{s} \notin \mathrm{dom}(\Delta) \text{ implies} & (\Gamma,\Delta\cdot s:?(U);S) \stackrel{s?\langle v\rangle}{\longrightarrow} (\Gamma,\Delta\cdot s:S) \\
\overline{s} \notin \mathrm{dom}(\Delta) \text{ implies} & (\Gamma,\Delta\cdot s:\oplus\{l_i:S_i\}_{i\in I}) \stackrel{s\oplus l_k}{\longrightarrow} (\Gamma,\Delta\cdot s:S_k) \\
\overline{s} \notin \mathrm{dom}(\Delta) \text{ implies} & (\Gamma,\Delta\cdot s:\&\{l_i:S_i\}_{i\in I}) \stackrel{s\&l_k}{\longrightarrow} (\Gamma,\Delta\cdot s:S_k) \\
\Delta \longrightarrow \Delta' \text{ implies} & (\Gamma,\Delta) \stackrel{\tau}{\longrightarrow} (\Gamma,\Delta')
\end{array}$$

Fig. 5. Labelled transition rules for environments.

The first rule says that reception of a message via a is possible only when a is input-typed ($\mathtt{i}$-mode) and its queue is present ($a \in \Delta$). The second is dual, saying that an output at a is possible only when a has $\mathtt{o}$-mode and no queue exists. Similarly for a bound output action. The two session output rules ($\ell = s!\langle v\rangle$ and $s!(a)$) are the standard value output and a scope opening rule. The next is for value input. Label input and output are defined similarly. Note that we send and receive only a shared channel which has $\mathtt{o}$-mode. This is because a new accept should not be created without its queue in the same location. The final rule ($\ell = \tau$) follows the reduction rules defined before Proposition 2.1. The LTS

omits delegations since it is not necessary in the present inquiry (due to the definition of localisation, see the following paragraph).

Write $\bowtie$ for a symmetric and transitive closure of $\longrightarrow$ over linear environments. We say a relation on typed processes is a *typed relation* if, whenever it relates two typed processes, we have $\Gamma \vdash P_1 \triangleright \Delta_1$ and $\Gamma \vdash P_2 \triangleright \Delta_2$ and $\Delta_1 \bowtie \Delta_2$ under Γ. We write $\Gamma \vdash P_1 \triangleright \Delta_1 \mathscr{R} P_2 \triangleright \Delta_2$ if $(\Gamma \vdash P_1 \triangleright \Delta_1, \Gamma \vdash P_2 \triangleright \Delta_2)$ are in a typed relation $\mathscr{R}$. Further we often leave the environments implicit, writing simply $P_1 \mathscr{R} P_2$.

Localisation and Bisimulation. Our bisimulation is a typed relation over processes which are *localised*, in the sense that they are equipped with all necessary local queues. We say an environment Δ is *delegation-free* if it contains types which are generated by deleting S from value type T in the syntax of types defined in § 2.3 (i.e. either $!(S);S'$ or $?(S);S'$ does not appear in Δ). Similarly for Γ. Now let P be closed and $\Gamma \vdash P \triangleright \Delta$ where Γ and Δ are delegation-free (note that P can perform delegations at hidden channels by $\langle \mathsf{Local} \rangle$). Then we say P is *localised* w.r.t. Γ, Δ if (1) For each $s : S \in \mathsf{dom}(\Delta)$, $s \in \Delta$; and (2) if $\Gamma(a) = \mathtt{i}\langle S \rangle$, then $a \in \Delta$. Being localised means that a process owns all necessary queues as specified in environments. We further say *P is localised* if it is so for a suitable pair of environments.

For example, $s?(x);s!\langle x+1\rangle;\mathbf{0}$ is not localised, while $s?(x);s!\langle x+1\rangle;\mathbf{0} \mid s[\mathtt{i}:\vec{h}_1, \mathtt{o}:\vec{h}_2]$ is localised. Similarly, $a(x).P$ is not localised, but $a(x).P \mid a[\vec{s}]$ is. By composing buffers at appropriate channels, any typable closed process can become localised. If P is localised w.r.t. (Γ, Δ) then $P \longrightarrow Q$ implies Q is localised w.r.t. (Γ, Δ), since queues always stay. We can now introduce the reduction congruence and the asynchronous bisimilarity.

Definition 3.1 (Reduction Congruence). We write $P \downarrow a$ if $P \equiv (\nu \vec{n})(\overline{a}\langle s \rangle \mid R)$ with $a \notin \vec{n}$. Similarly we write $P \downarrow s$ if $P \equiv (\nu \vec{n})(s[\mathtt{o}:h\cdot\vec{h}] \mid R)$ with $s \notin \vec{n}$. $P \Downarrow n$ means $\exists P'.P \twoheadrightarrow P' \downarrow n$. A typed relation $\mathscr{R}$ is *reduction congruence* if it is a congruent and satisfies the following conditions for each $P_1 \mathscr{R} P_2$ whenever they are localised w.r.t. their given environments.

1. $P_1 \Downarrow n$ iff $P_2 \Downarrow n$.
2. Whenever $\Gamma \vdash P_1 \triangleright \Delta_1 \mathscr{R} P_2 \triangleright \Delta_2$ holds, $P_1 \twoheadrightarrow P_1'$ implies $P_2 \twoheadrightarrow P_2'$ such that $\Gamma \vdash P_1' \triangleright \Delta_1' \mathscr{R} P_2' \triangleright \Delta_2'$ holds with $\Delta_1' \bowtie \Delta_2'$ and the symmetric case.

The maximum reduction congruence which is not a universal relation exists [9] which we call *reduction congruency*, denoted by $\cong$.

Definition 3.2 (Asynchronous Session Bisimulation). A typed relation $\mathscr{R}$ over localised processes is a *weak asynchronous session bisimulation* or often a *bisimulation* if, whenever $\Gamma \vdash P_1 \triangleright \Delta_1 \mathscr{R} P_2 \triangleright \Delta_2$, the following two conditions holds: (1) $\Gamma \vdash P_1 \triangleright \Delta_1 \xrightarrow{\ell} \Gamma' \vdash P_1' \triangleright \Delta_1'$ implies $\Gamma \vdash P_2 \triangleright \Delta_2 \stackrel{\hat{\ell}}{\Longrightarrow} \Gamma' \vdash P_2' \triangleright \Delta_2'$ such that $\Gamma' \vdash P_1' \triangleright \Delta_1' \mathscr{R} P_2' \triangleright \Delta_2'$ with $\Delta_1' \bowtie \Delta_2'$ holds and (2) the symmetric case of (1). The maximum bisimulation exists which we call *bisimilarity*, denoted by $\approx$. We sometimes leave environments implicit, writing e.g. $P \approx Q$.

We extend $\approx$ to possibly non-localised closed terms by relating them when their minimal localisations are related by $\approx$ (given $\Gamma \vdash P \triangleright \Delta$, its *minimal localisation* adds empty queues to P for the input shared channels in Γ and session channels in Δ that are missing their queues). Further $\approx$ is extended to open terms in the standard way [9].

3.2 Properties of Asynchronous Session Bisimilarity

Characterisation of Reduction Congruence. This subsection studies central properties of asynchronous session semantics. We first show that the bisimilarity coincides with the naturally defined reduction-closed congruence [9], given below.

Theorem 3.3 (Soundness and Completeness). $\approx = \cong$.

The soundness ($\approx \subset \cong$) is by showing $\approx$ is congruent. The most difficult case is a closure under parallel composition, which requires to check the side condition $\Delta_1' \bowtie \Delta_2'$ for each case. The completeness ($\cong \subset \approx$) follows [6, § 2.6] where we prove that every external action is definable by a testing process, see Appendix C.1.

Asynchrony and Session Determinacy. Let us call ℓ an *output action* if ℓ is one of $\overline{a}\langle s\rangle, \overline{a}(s), s!\langle v\rangle, s!(a), s \oplus l$; and an *input action* if ℓ is one of $a\langle s\rangle, s?\langle v\rangle, s\&l$. In the following, the first property says that we can delay an output arbitrarily, while the second says that we can always immediately perform a (well-typed) input.

Lemma 3.4 (Input and Output Asynchrony). *Suppose* $\Gamma \vdash P \triangleright \Delta \stackrel{\ell}{\Longrightarrow} P' \triangleright \Delta'$.

- (input advance) *If* ℓ *is an input action, then* $\Gamma \vdash P \triangleright \Delta \stackrel{\ell}{\longrightarrow\!\!\Longrightarrow} P' \triangleright \Delta'$.
- (output delay) *If* ℓ *is an output action, then* $\Gamma \vdash P \triangleright \Delta \Longrightarrow\!\!\stackrel{\ell}{\longrightarrow} P' \triangleright \Delta'$.

The asynchronous environment interaction on the session buffers enables input actions to happen before multi-internal steps and output actions to happen after multi-internal steps.

Following [21], we define determinacy and confluence. Below and henceforth we often omit the environments in typed transitions.

Definition 3.5 (Determinacy). We say $\Gamma' \vdash Q \triangleright \Delta'$ is *derivative* of $\Gamma \vdash P \triangleright \Delta$ if there exists $\vec{\ell}$ such that $\Gamma \vdash P \triangleright \Delta \stackrel{\vec{\ell}}{\Longrightarrow} \Gamma' \vdash Q \triangleright \Delta'$. We say $\Gamma \vdash P \triangleright \Delta$ is *determinate* if for each derivative Q of P and action ℓ, if $Q \stackrel{\ell}{\longrightarrow} Q'$ and $Q \stackrel{\hat{\ell}}{\Longrightarrow} Q''$ then $Q' \approx Q''$.

We then extend the above notions to session communications.

Definition 3.6 (Session Determinacy). Let us write $P \stackrel{\ell}{\longrightarrow}_s Q$ if $P \stackrel{\ell}{\longrightarrow} Q$ where if $\ell = \tau$ then it is generated without using [Request1], [Request2], [Accept], [Areq] nor [Amsg] in Figure 3 (i.e. a communication is performed without arrival predicates or accept actions). We extend the definition to $\stackrel{\vec{\ell}}{\Longrightarrow}_s$ and $\stackrel{\hat{\ell}}{\Longrightarrow}_s$ etc. We say P is *session determinate* if P is typable and localised and if $\Gamma \vdash P \triangleright \Delta \stackrel{\vec{\ell}}{\Longrightarrow} Q \triangleright \Delta'$ then $\Gamma \vdash P \triangleright \Delta \stackrel{\vec{\ell}}{\Longrightarrow}_s Q \triangleright \Delta'$. We call such Q a *session derivative* of P.

We define $\ell_1 \lfloor \ell_2$ as (1) $\overline{a}\langle s\rangle$ if $\ell_1 = \overline{a}(s')$ and $s' \in \mathsf{bn}(\ell_2)$; (2) $s!\langle s'\rangle$ if $\ell_1 = s!(s')$ and $s' \in \mathsf{bn}(\ell_2)$; (3) $s!\langle a\rangle$ if $\ell_1 = s!(a)$ and $a \in \mathsf{bn}(\ell_2)$; and otherwise ℓ_1. We write that $l_1 \bowtie l_2$ when $l_1 \neq l_2$ and if l_1, l_2 are input actions then $\mathsf{sbj}(l_1) \neq \mathsf{sbj}(l_2)$.

Definition 3.7 (Confluence). We say $\Gamma \vdash P \triangleright \Delta$ is *confluent* if for each derivative Q of P and actions ℓ_1, ℓ_2 such that $\ell_1 \bowtie \ell_2$, (i) if $Q \stackrel{\ell}{\longrightarrow} Q_1$ and $Q \stackrel{\ell}{\Longrightarrow} Q_2$, then $Q_1 \Longrightarrow Q_1'$ and $Q_2 \Longrightarrow Q_2' \approx Q_1'$; and (ii) if $Q \stackrel{\ell_1}{\longrightarrow} Q_1$ and $Q \stackrel{\ell_2}{\Longrightarrow} Q_2$, then $Q_1 \stackrel{\widehat{\ell_2 \lfloor \ell_1}}{\Longrightarrow} Q_1'$ and $Q_2 \stackrel{\widehat{\ell_1 \lfloor \ell_2}}{\Longrightarrow} Q_2' \approx Q_1'$.

Lemma 3.8. *Let* P *be session determinate and* $\Gamma \vdash P \Longrightarrow Q \triangleright \Delta$. *Then* $P \approx Q$.

Theorem 3.9 (Session Determinacy). *Let* P *be session determinate. Then* P *is determinate and confluent.*

The following relation is used to prove the event-based optimisation.

Definition 3.10 (Determinate Upto-expansion Relation). Let $\mathscr{R}$ be a symmetric, typed relation such that if $\Gamma \vdash P \triangleright \Delta \ \mathscr{R} \ Q \triangleright \Delta$ and (1) P, Q are determinate; (2) If $\Gamma \vdash P \triangleright \Delta \xrightarrow{l} \Gamma' \vdash P'' \triangleright \Delta''$ then $\Gamma \vdash Q \triangleright \Delta \stackrel{l}{\Longrightarrow} \Gamma' \vdash Q' \triangleright \Delta'$ and $\Gamma' \vdash P'' \triangleright \Delta'' \Longrightarrow \Gamma' \vdash P' \triangleright \Delta'$ with $\Gamma' \vdash P' \triangleright \Delta' \mathscr{R} \ Q' \triangleright \Delta'$; and (3) the symmetric case. Then we call $\mathscr{R}$ a *determinate upto-expansion relation*, or often simply *upto-expansion relation*.

Lemma 3.11. *Let $\mathscr{R}$ be an upto-expansion relation. Then $\mathscr{R} \subset \approx$.*

The proof is by showing $\Longrightarrow \mathscr{R} \Longleftarrow$ with $\Longrightarrow$ determinate is a bisimulation.

3.3 Examples of Induced Equations: Permutation and Asynchronous Events

This subsection shows example equations (algebra of processes) pertaining to properties of permutations of actions studied in § 3.2 and the semantic effects of `arrival` predicates. Significant equations induced by a behavioural equivalence offer insights on the nature of process behaviours we are concerned with, and play a key role in reasoning about processes.

We use the examples from § 1. Let $R_i = s_i\,[\mathtt{i}:\vec{h}_i, \mathtt{o}:\vec{h}'_i]$ for some $\vec{h}_i, \vec{h}'_i$ and assume $s_1 \neq s_2$.
(1) Input and Output Permutations. The two actions at different channels are permutable up to $\approx$, i.e. $s_1?(x);s_2?(y);P \mid R_1 \mid R_2 \approx s_2?(y);s_1?(x);P \mid R_1 \mid R_2$. Similarly for two outputs: $s_1!\langle v_1\rangle;s_2!\langle v_2\rangle;P \mid R_1 \mid R_2 \approx s_2!\langle v_2\rangle;s_1!\langle v_1\rangle;P \mid R_1 \mid R_2$. However, an input and an output are not permutable: $s_1?(x);s_2!\langle v_2\rangle;P \mid R_1 \mid R_2 \not\approx s_2!\langle v_2\rangle;s_1?(x);P \mid R_1 \mid R_2$.
(2) Input and Output Ordering. Two actions at the same session have the ordering, hence: $s_1?(x);s_1?(y);P \mid R_1 \not\approx s_1?(y);s_1?(x);P \mid R_1$ for inputs, $s_1!\langle v_1\rangle;s_1!\langle v_2\rangle;P \mid R_1 \not\approx s_1!\langle v_2\rangle;s_1!\langle v_1\rangle;P \mid R_1$ for outputs and $s_1!\langle v_1\rangle;s_2?(x_2);P \not\approx s_2?(x_2);s_1!\langle v_1\rangle;P$.
(3) Non-Local Semantics. In the literature [4, 5, 20], the asynchronous session types are modelled by the *two end-point queues* without distinction between input and output entries in queues. We call this semantics *non-local* since the output process directly puts the value into the input queue. The transition relation for non-local semantics is defined by replacing the output and selection rules in Figure 4 (see Appendix D.3 for full details) to:

$$\langle \mathsf{Out}_n \rangle \ \ s!\langle v\rangle;P \xrightarrow{s!\langle v\rangle} P \qquad \langle \mathsf{Sel}_n \rangle \ \ s \oplus l;P \xrightarrow{s\oplus l} P$$

We write the induced bisimilarity $\approx_2$. The same (in)equalities hold except permutation of outputs, e.g. $s_1!\langle v_1\rangle;s_2!\langle v_2\rangle;P \mid R_1 \mid R_2 \not\approx_2 s_2!\langle v_2\rangle;s_1!\langle v_1\rangle;P \mid R_1 \mid R_2$.
(4) Arrival Predicates. Let $Q = \mathtt{if}\ e\ \mathtt{then}\ P_1\ \mathtt{else}\ P_2$ with $P_1 \not\approx P_2$. If the syntax does not include arrival predicates, we have $Q \mid s[\mathtt{i}:\emptyset] \mid \overline{s}[\mathtt{o}:v] \approx Q \mid s[\mathtt{i}:v] \mid \overline{s}[\mathtt{o}:\emptyset]$. In the presence of the arrival predicate, we have $Q \mid s[\mathtt{i}:\emptyset] \mid \overline{s}[\mathtt{o}:v] \not\approx Q \mid s[\mathtt{i}:v] \mid \overline{s}[\mathtt{o}:\emptyset]$ with $e = \mathtt{arrived}\ s$. However we have: $\mathtt{if}\ \mathtt{arrived}\ s\ \mathtt{then}\ P\ \mathtt{else}\ P \approx P$.
(5) Arrive Inspection Ordering. We give a basic equation for (a simplified form of) the standard event loop, handling events by non-blocking handlers. We use recursive equations of agents for legibility, which can be easily encoded into recursions.

$$P_1 = \mathtt{if}\ \mathtt{arrived}\ s_1\ \mathtt{then}\ (s_1?(x).R_1);P_2\ \mathtt{else}\ \mathtt{if}\ \mathtt{arrived}\ s_2\ \mathtt{then}\ (s_2?(x).R_2);P_1\ \mathtt{else}\ P_1$$
$$P_2 = \mathtt{if}\ \mathtt{arrived}\ s_2\ \mathtt{then}\ (s_2?(x).R_2);P_1\ \mathtt{else}\ \mathtt{if}\ \mathtt{arrived}\ s_1\ \mathtt{then}\ (s_1?(x).R_1);P_2\ \mathtt{else}\ P_2$$

where we assume well-typedness and each of $R_{1,2}$, under any closing substitution and localisation, is determinate and reaches $\mathbf{0}$ after a series of outputs and τ-actions. The sequencing $(s_1?(x).R_1);P_2$ denotes the process obtained by replacing each $\mathbf{0}$ in R_1 with P_2. Process P_1 tries to process messages at s_1 and s_2 repeatedly in this order without blocking, while P_2 does the same in the reverse order (in practice, each of $R_{1,2}$ would use the *typecase*, cf. §4 later, to process different segments of a single session). We can then show $P_1 \approx P_2$. The proof is by the up-to-expansion relation consisting of arbitrary localisations of such pairs. Later we use a generalised form of this equation for the Lauer-Needham transform.

4 Lauer-Needham Transform

In an early work [14], Lauer and Needham observed that a concurrent program may be written equivalently either in a thread-based programming style (with shared memory primitives) or in an event-based style (with a single-threaded event loop processing messages sequentially with non-blocking handlers). Following this framework and using high-level asynchronous event primitives such as *selectors* [17] for the event-based style, many studies compare these two programming styles, often focusing on performance of server architectures (see [12, § 6] for recent studies on event programming). These implementations implicitly or explicitly assume a *transformation* from a program written in the thread-based style, especially those which generate a new thread for each service request (as in thread-based web servers), to its *equivalent* event-based program, which treats concurrent services using a single threaded event-loop (as in event-based web servers). However the precise semantic effects of such a transformation nor the exact meaning of the associated "equivalence" has not been clarified.

We study the semantic effects of such a transformation using the asynchronous session bisimulation. We first introduce a formal mapping from a thread-based process to their event-based one [14]. Assuming a server process whose code creates fresh threads at each service invocation, the key idea is to decompose this whole code into distinct smaller code segments each handling the part of the original code starting from a blocking action. Such a blocking action is represented as reception of a message (input or branching). Then a single global event-loop can treat each message arrival by processing the corresponding code segment combined with an environment, returning to inspect the content of event/message buffers. We first stipulate a class of processes which we consider for our translation. Below $*a(x);P$ denotes an *input replication* abbreviating $\mu X.a(x).(P|X)$.

Definition 4.1 (Server). A *simple server at a* is a closed process $*a(x).P$ with a typing of form $a:\mathtt{i}\langle S\rangle, b_1:\mathtt{o}\langle S_1\rangle, .., b_n:\mathtt{o}\langle S_n\rangle$ where P is sequential (i.e. contains no parallel composition $|$) and is determinate under any closing substitution and any localisation. A simple server is often considered with its localisation with an empty queue $a[\varepsilon]$.

A server spawns an unbounded number of threads as it receives session requests repeatedly. Each thread may initiate other sessions with outside, and its interactions may involve delegations and name passing. But a server does *not* involve accesses to local state among threads. A practical example is a web-server which only serves static web pages. Given a server $*a(w:S);P \mid a[\varepsilon]$, its translation, which we call *Lauer-Needham transform* or *LN-transform* for short, is written $\mathscr{LN}[\![*a(w:S);P \mid a[\varepsilon]]\!]$.

We outline the key ideas through examples (the full mapping is non-trivial and given in Appendix F). First, $\mathscr{LN}[\![*a(w:S);P]\!]$ consists of the following key elements:

1. An *event loop* $\mathsf{Loop}\langle o,q\rangle$ repeats the standard *event-loop*, inspecting its *selector queue* named q (see below), *selects* a message from a message arrived and processes it by sending it to the corresponding *code block* (see below).
2. A *selector queue* $q\langle a,c_0\rangle$ whose initial element is $\langle a,c_0\rangle$. This data says: "if a message comes at a, jump to the code block (CPS procedure) whose subject is c_0".
3. A collection of *code blocks* $\mathsf{CodeBlocks}\langle a,o,q,\vec{c}\rangle$, CPS procedures handling in-coming messages. A code block originates from a *blocking subterm*, i.e. a subterm starting from an input or a branching.

The process uses a standard "select" primitive represented as a process, called *selector* [12]. It stores *a collection of session channels*, with each channel associated with an environment, binding variables to values. It then picks up one of them at which a message

arrives, receives that message via that channel and has it be processed by the corresponding code block (which may alter the environment). Finally it stores the session and the associated environment back in the collection, and moves to the next iteration. Since a selector should handle channels of different types, it uses the `typecase` construct from [12]. $\texttt{typecase}\ k\ \texttt{of}\ \{(x_i : T_i)\, P_i\}_{i\in I}$ takes a session endpoint k and a list of cases $(x_i : T_i)$, each binding the free variable x_i of type pattern T_i in P_i. Its reduction is defined as:

$$\texttt{typecase}\ s\ \texttt{of}\ \{(x_i : T_i)\, P_i\}_{i\in I} \mid s\,[S, \texttt{i} : \vec{h}, \texttt{o} : \vec{h'}] \longrightarrow P_i\{s/x_i\} \mid s\,[S, \texttt{i} : \vec{h}, \texttt{o} : \vec{h'}]$$

where there exists $i \in I$ such that $(\forall j < i . T_j \not\leq S \wedge T_i \leq S)$ where $\leq$ denotes a subtyping relation. The `typecase` constructs matches the session type of the tested channel to a session type in its defined list and proceeds with the corresponding process. For the matching to take place, session endpoint configuration syntax is extended with the runtime session typing [12]. The selector operations are defined by the following reduction semantics.

$$\begin{array}{l}
\texttt{new selector}\ r\ \texttt{in}\ P \longrightarrow (\nu\, r)(P \mid \mathsf{sel}\langle r, \varepsilon\rangle) \qquad \texttt{register}\langle s', r\rangle; P \mid \mathsf{sel}\langle r, \vec{s}\rangle \longrightarrow P \mid \mathsf{sel}\langle r, \vec{s}\cdot s'\rangle \\
\texttt{let}\ x = \texttt{select}(r)\ \texttt{in typecase}\ x\ \texttt{of}\ \{(x_i : T_i) : P_i\}_{i\in I} \mid \mathsf{sel}\langle r, s'\cdot\vec{s}\rangle \mid s'\,[S, \texttt{i} : \vec{h}] \\
\qquad\qquad \longrightarrow P_i\{s'/x_i\} \mid \mathsf{sel}\langle r, \vec{s}\rangle \mid s'\,[S, \texttt{i} : \vec{h}] \quad (\vec{h} \neq \varepsilon) \\
\texttt{let}\ x = \texttt{select}(r)\ \texttt{in typecase}\ x\ \texttt{of}\ \{(x_i : T_i) : P_i\}_{i\in I} \mid \mathsf{sel}\langle r, s'\cdot\vec{s}\rangle \mid s'\,[\texttt{i} : \varepsilon] \\
\qquad\qquad \longrightarrow \texttt{let}\ x = \texttt{select}(r)\ \texttt{in typecase}\ x\ \texttt{of}\ \{(x_i : T_i) : P_i\}_{i\in I} \mid \mathsf{sel}\langle r, \vec{s}\cdot s'\rangle \mid s'\,[\texttt{i} : \varepsilon]
\end{array}$$

where in the third line S and T_i satisfy the condition for `typecase` in the reduction rule. Operator $\texttt{new selector}\ r\ \texttt{in}\ P$ (binding r in P) creates a new selector $\mathsf{sel}\langle r, \varepsilon\rangle$, named r and with the empty queue ε. Operator $\texttt{register}\langle s', r\rangle; P$ registers a session channel s to r, adding s' to the original queue $\vec{s}$. The next `let` retrieves a registered session and checks the availability to test if an event has been triggered. If so, find the match of the type of s' among $\{T_i\}$ and select P_i; if not, the next session is tested. As proved in [12], these primitives are encodable in the original calculus augmented with `typecase`. The bisimulations and their properties (such as congruency of $\approx$) remain unchanged.

Example 4.1 (Lauer-Needham Transform). As an example of a server, consider:

$$P \;=\; *a(x); x?(y).x!\langle y+1\rangle; x?(z).x!\langle y+z\rangle; \mathbf{0} \mid a[\varepsilon]$$

This process has the session type $?(\mathsf{nat}); !(\mathsf{nat})?(\mathsf{nat}); !(\mathsf{nat})$ at a which can be read: *a process should first expect to receive* ? *a message of type* nat *and send* ! *it, then to receive (again* ?*) a* nat*, and finish by sending* ! *a result.* We extract the blocking subterms from this process as follows.

Blocking Process	Type at Blocking Prefix
$a(x).x?(y).x!\langle y+1\rangle x?(z).x!\langle y+z\rangle; \mathbf{0}$	$\texttt{i}\langle ?(\mathsf{nat}); !(\mathsf{nat}); ?(\mathsf{nat}); !(\mathsf{nat})\rangle$
$x?(y).x!\langle y+1\rangle x?(z).x!\langle y+z\rangle; \mathbf{0}$	$?(\mathsf{nat}); !(\mathsf{nat}); ?(\mathsf{nat}); !(\mathsf{nat})$
$x?(z).x!\langle y+z\rangle; \mathbf{0}$	$?(\mathsf{nat}); !(\mathsf{nat})$

These blocking processes are translated into *code blocks*, denoted CodeBlocks, given as:

$$\begin{array}{l}
*c_0(y); a(x).\texttt{update}(y,x,x); \texttt{register}\ \langle sel, x, y, c_1\rangle; \overline{o} \mid \\
*c_1(x,y); x?(z); \texttt{update}(y,z,z); x!\langle [\![z]\!]_y + 1\rangle; \texttt{register}\ \langle sel, x, y, c_2\rangle; \overline{o} \mid \\
*c_2(x,y); x?(z'); \texttt{update}(y,z',z'); x!\langle [\![z]\!]_y + [\![z']\!]_y\rangle; \overline{o}
\end{array}$$

which is used for processing each message. Above, the operation $\texttt{update}(y,x,x);$ updates an environment, while `register` stores the blocking session channel, the associated continuation c_i and the current environment y in a selector queue sel.

Finally, using these code blocks, the main event-loop denoted Loop, is given as:

$$\begin{array}{l}\mathtt{Loop} = *o.\mathtt{let}\ (x,y,z) = \mathtt{select\ from}\ \mathit{sel}\ \mathtt{in\ typecase}\ x\ \mathtt{of}\ \{\\ \qquad \mathtt{i}\langle ?(\mathsf{nat});!(\mathsf{nat});?(\mathsf{nat});!(\mathsf{nat})\rangle\ :\ \mathtt{new}\ y:\mathtt{env}\ \mathtt{in}\ \overline{z}(y)\\ \qquad ?(\mathsf{nat});!(\mathsf{nat});?(\mathsf{nat});!(\mathsf{nat})\quad :\ \overline{z}(x,y)\\ \qquad ?(\mathsf{nat});!(\mathsf{nat})\quad :\ \overline{z}(x,y)\}\end{array}$$

Above `select from` *sel* `in` selects a message from the selector queue *sel*, and treats it in P. The `new` construct creates a new environment y. The `typecase` construct then branches into different processes depending on the session of the received message, and dispatch the task to each code block.

Because a server does not allow, by construction, its internal shares state to be accessed by the threads it spawns, it is currently stateless.[2] Hence we have:

Lemma 4.2. *$*a(w:S);R \mid a[\varepsilon]$ is confluent.*

Lemma 4.3. *Let $P \stackrel{def}{=} \mu X.\mathtt{let}\ x = \mathtt{select}(r)\ \mathtt{in\ typecase}\ x\ \mathtt{of}\ \{(x_i:T_i):R_i;X\}_{i\in I}$ where each R_i is determinate and reaches $\mathbf{0}$ after a sequence of outputs and τ-actions as well as a single input/branching action at x_i. The sequencing $R_i;X$ is defined as in Example 3.3(5). Then, assuming typability, we have $P \mid \mathsf{sel}\langle r, \vec{s}_1\cdot s_1'\cdot s_2'\cdot\vec{s}_2\rangle \approx P \mid \mathsf{sel}\langle r, \vec{s}_1\cdot s_2'\cdot s_1'\cdot\vec{s}_2\rangle$.*

The above lemma substantiates a generalisation of the observations behind Example 3.3(5), distilling the nature of event-driven programming. With this lemma, we can permute the session channels in a selector queue, while keeping the same behaviours, which is possible w.r.t. bisimilarity because of the stateless nature of a server. As we shall see below, this selector's behaviour ties together threaded and event programs because there is no difference between which event (resp. thread) is selected to be executed first.

Theorem 4.4 (Semantic Preservation). *Let $*a(w:S);R \mid a[\varepsilon]$ be a simple server. Then $*a(w:S);P \mid a[\varepsilon] \approx \mathscr{LN}[\![a(w:S);P \mid a[\varepsilon]]\!]$.*

The proof of the above theorem constructs a determinate upto-expansion relation, cf. Definition 3.10 and Lemma 3.11, which contains each process pair that has all the parallel processes on a blocking prefix for the threaded server and starts from the Loop process for the thread eliminated process, with arbitrary localisations. We show the conditions needed Definition 3.10 by using Lemmas G.5. We conclude the proof through Lemma 3.11. For details of the proof, see Appendix F.

5 Discussions

Comparisons with Asynchronous/Synchronous Calculi. We briefly compared the present asynchronous bisimulation to related ones in Example 3.3 in § 3. Below we report more comprehensive comparisons, clarifying the relationship between (1) the session-typed asynchronous π-calculus [7] without queues ($\approx_a$, the asynchronous version of the labelled transition relation for the asynchronous π-calculus), (2) the session-typed synchronous π-calculus [8, 23] without queues ($\approx_s$), (3) the asynchronous session π-calculus with two end-point queues without IO queues [4, 5, 20] ($\approx_2$), see Example 3.3; and (4) the asynchronous session π-calculus with two end-point IO-queues ($\approx$), i.e. the one developed in this paper. See Appendix D for the full definitions and proofs.

[2] The transform easily extends to the situation where threads share state, though its behavioural justification takes a different form.

The following figure summarises distinguishing examples. Non-Blocking Input/Output means inputs/outputs on different channels, while the Input/Output Order-Preserving means that the messages will be received/delivered preserving the order. The final table explains whether Lemma 3.4 (1) (input advance) or (2) (output delay) is satisfied or not. If not, we place a counterexample.

	Non-Blocking Input	Non-Blocking Output
(1)	$s_1?(x);s_2?(y);P \approx_a s_2?(y);s_1?(x);P$	$\overline{s_1}\langle v\rangle \mid \overline{s_2}\langle w\rangle \mid P \approx_a \overline{s_1}\langle w\rangle \mid \overline{s_2}\langle v\rangle \mid P$
(2)	$s_1?(x);s_2?(y);P \not\approx_s s_2?(y);s_1?(x);P$	$s_1!\langle v\rangle;s_2!\langle w\rangle;P \not\approx_s s_2!\langle w\rangle;s_1!\langle v\rangle;P$
(3)	$s_1?(x);s_2?(y);P \mid s_1[\varepsilon] \mid s_2[\varepsilon] \approx_2$ $s_2?(y);s_1?(x);P \mid s_1[\varepsilon] \mid s_2[\varepsilon]$	$s_1!\langle v\rangle;s_2!\langle w\rangle;P \mid s_1[\varepsilon] \mid s_2[\varepsilon] \not\approx_2$ $s_2!\langle w\rangle;s_1!\langle v\rangle;P \mid s_1[\varepsilon] \mid s_2[\varepsilon]$
(4)	$s_1?(x);s_2?(y);P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \approx$ $s_2?(y);s_1?(x);P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon]$	$s_1!\langle v\rangle;s_2!\langle w\rangle;P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \approx$ $s_2!\langle w\rangle;s_1!\langle v\rangle;P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon]$

	Input Order-Preserving	Output Order-Preserving
(1)	$s?(x);s?(y);P \approx_a s?(y);s?(x);P$	$\overline{s}\langle v\rangle \mid \overline{s}\langle w\rangle \mid P \approx_a \overline{s}\langle w\rangle \mid \overline{s}\langle v\rangle \mid P$
(2)	$s?(x);s?(y);P \not\approx_s s?(y);s?(x);P$	$s!\langle v\rangle;s!\langle w\rangle;P \not\approx_s s!\langle w\rangle;s!\langle v\rangle;P$
(3)	$s?(x);s?(y);P \mid s[\varepsilon] \not\approx_2$ $s?(x);s?(y);s?(x);P \mid s[\varepsilon]$	$s!\langle v\rangle;s!\langle w\rangle;P \mid s[\varepsilon] \not\approx_b$ $s!\langle w\rangle;s!\langle v\rangle;P \mid s[\varepsilon]$
(4)	$s?(x);s?(y);P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \not\approx$ $s?(x);s?(y);P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon]$	$s!\langle v\rangle;s!\langle w\rangle;P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \not\approx$ $s!\langle w\rangle;s!\langle v\rangle;P \mid [s_1,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon] \mid [s_2,\mathtt{i}:\varepsilon,\mathtt{o}:\varepsilon]$

	Lemma 3.4 (1)	Lemma 3.4 (2)
(1)	yes	yes
(2)	$(\nu\, s)(s!\langle v\rangle;s'?(x);\mathbf{0} \mid s?(x);\mathbf{0})$	$(\nu\, s)(s!\langle v\rangle;s'!\langle v'\rangle;\mathbf{0} \mid s'?(x);\mathbf{0})$
(3)	yes	$s!\langle v\rangle;\, s'?(x);\mathbf{0} \mid s'[v']$
(4)	yes	yes

Another technical interest is the effects of the `arrived` predicate on these combinations. We define the synchronous and asynchronous π-calculi augmented with the `arrived` predicate and local buffers. For the asynchronous π-calculus, we add $a[\vec{h}]$ and `arrived` a in the syntax, and define the following rules for input and outputs.

$$\overline{a}\langle v\rangle \xrightarrow{\overline{a}\langle v\rangle} \mathbf{0} \qquad a[\vec{h}] \xrightarrow{a\langle h\rangle} a[\vec{h}\cdot h] \qquad\qquad \texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, Q | a[\varepsilon] \xrightarrow{\tau} Q \mid a[\varepsilon]$$

$$a?(x).P \mid a[\vec{h_1}\cdot h_i\cdot \vec{h_2}] \longrightarrow P\{h_i/x\} \mid a[\vec{h_1}\cdot\vec{h_2}] \qquad \texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, Q | a[h\cdot\vec{h}] \xrightarrow{\tau} P \mid a[h\cdot\vec{h}]$$

The above definition precludes the order preservation as the property of transport, but still keeps the non-blocking property as in the asynchronous π-calculus. The synchronous version is similarly defined by setting the buffer size to be one. The non-local version is defined just by adding `arrived` predicate.

	With `arrived`	Without `arrived`
(1)	$\texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:\varepsilon] \mid \overline{s}\langle v\rangle$ $\not\approx \texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:v]$	$\texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:\varepsilon] \mid \overline{s}\langle v\rangle$ $\approx \texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:v]$
(2)	$\texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\varepsilon] \mid \overline{s}\langle v\rangle;\mathbf{0}$ $\not\approx \texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[v]$	$\texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\varepsilon] \mid \overline{s}\langle v\rangle;\mathbf{0}$ $\approx \texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[v]$
(3)	$\texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:\varepsilon] \mid \overline{s}\langle v\rangle;\mathbf{0}$ $\not\approx \texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:v]$	$\texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:\varepsilon] \mid \overline{s}\langle v\rangle;\mathbf{0}$ $\approx \texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:v]$
(4)	$\texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:\varepsilon] \mid s[\mathtt{o}:v]$ $\not\approx \texttt{if arrived}\, s\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:v] \mid s[\mathtt{o}:\varepsilon]$	$\texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:\varepsilon] \mid s[\mathtt{o}:v]$ $\approx \texttt{if}\, e\, \texttt{then}\, P\, \texttt{else}\, Q \mid s[\mathtt{i}:v] \mid s[\mathtt{o}:\varepsilon]$

Fig. 6. Arrive inspection behaviour in synchronous/asynchronous calculi.

Figure 6 summarises the results which incorporate the `arrived` predicate. Interestingly in all of the calculi (1–4), the same examples as in Example 3.3(4), which separate semantics with/without the `arrived`, are effective. The IO queues provide non-blocking

inputs and outputs, while preserving the input/output ordering, which distinguishes the present framework from other known semantics. As a whole, we observe that the present semantic framework is closer to the asynchronous bisimulation (1) $\approx_a$ augmented with order-preserving nature per session. Its key properties arise from local, buffered session semantics and typing. We have also seen the semantic significance of the `arrived` predicates, which enables processes to observe the effects of fine-grained synchronisations.

Related Work. Some of the key proof methods of our work draw their ideas from [21], which study an extension of the confluence theory on the π-calculus. They apply the theory to reason about the correctness of the distributed protocol which can be represented by constructing a collection of confluent processes. Our work differs in that we investigate the effect of asynchronous IO queues and its relationship to confluence.

The work [1] examines expressiveness of various messaging mediums by adding message bags (no ordering), stacks (LIFO policy) and message queues (FIFO policy) in the asynchronous π-calculus [7]. They show that the calculus with the message bags is encodable into the asynchronous π-calculus, but it is impossible to encode the message queues and stacks. Neither the effects of locality, queues, and typed transitions are studied. Further neither of [1, 21] treats event-based programming, including such examples as thread elimination nor performance analysis.

Programming constructs that can test the presence of actions or events are studied in the context of the Linda language [3] and CSP [15, 16]. The work [3] measures expressive powers between three variants of asynchronous Linda-like calculi, with a construct for inspecting the output in the tuple space, which is reminiscent of the *inp* predicate of Linda. The first calculus (called *instantaneous*) corresponds to (1) [7], the second one (called *ordered*) formalises emissions of messages to the tuple spaces, and the third one (called *unordered*) models unordered outputs in the tuple space by decomposing one messaging into two stages — emission from an output process and rendering from the tuple space. It shows that the instantaneous and ordered calculi are Turing powerful, while the unordered is not. The work [15] studies CSP with a construct that checks if a parallel process is able to perform an output action on a given channel. It studies operational and denotational semantics, demonstrating a significance to investigate event primitives using process calculi. A subsequent work [16] studies the expressiveness of its variants focussing on the full abstraction theorem of the trace equivalence. Due to the difference of the base calculi and the aims of the primitives, direct comparisons are difficult: for example, our calculi (1,2,3,4) are Turing powerful and we aim to examine properties and applications of the typed bisimilarity characterised by buffered sessions: on the other hand, the focus of [3] is a tuple space where our input/output order preserving examples (which treat different objects with the same session channel) cannot be naturally (and efficiently) defined. The same point applies for [15, 16]. As another difference, the nature of localities has not been considered either in [3, 15, 16] since no notion of a local or remote tuple or environment is defined. Further, neither large applications which include these constructs (§ 4), the equivalences as we treated, nor the performance analysis of the proposed primitives had been discussed in [3, 15, 16].

Using the confluence and determinacy guaranteed by session types, and through observations on the semantics of the arrive predicate, we have demonstrated that the theory is applicable, through the verification of the correctness of the Lauer-Needham transform. This well-known transform claims that threads and events are dual to each other. Our LN-transform and the asynchronous, buffered bisimulation provide a formal framework and reasoning mechanisms for the conversion from the former to latter and for establishing their equivalence. The asynchronous nature realised through IO message queues provides a precise analysis of local and eventful behaviours, found in major distributed transports

such as TCP. The benchmark results from high-performance clusters in Appendix H show that the throughput for the thread-eliminated Server implementations in Session Java (SJ) [12] exhibits higher throughput than the multithreaded Server implementations, justifying the effect of the type and semantic preserving LN-transformation.

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Table of Contents

A Appendix for Section 2

We give the definitions that were omitted from § 2 including the type case construct.

$$
\begin{array}{rll}
P \equiv Q \quad \text{if } P =_\alpha Q & & (\alpha\text{-renaming}) \\
P \mid \mathbf{0} \equiv P & & (\text{Idempotence}) \\
P \mid Q \equiv Q \mid P & & (\text{Commutativity}) \\
(P \mid P') \mid P'' \equiv P \mid (P' \mid P'') & & (\text{Associativity}) \\
(\nu\, a{:}\langle S\rangle)\, a[\varepsilon] \equiv \mathbf{0} & & (\text{Shared channels}) \\
(\nu\, a{:}\langle S\rangle)\, P \mid Q \equiv (\nu\, a{:}\langle S\rangle)\,(P \mid Q) & (a \notin \mathrm{fn}(Q)) & \\
(\nu\, s)\, \mathbf{0} \equiv \mathbf{0} & & (\text{Session channels}) \\
(\nu\, s)\,(s : [\varepsilon] \mid \overline{s} : [\varepsilon]) \equiv \mathbf{0} & & (\text{Session queues}) \\
(\nu\, s)\, P \mid Q \equiv (\nu\, s)\,(P \mid Q) & (s \notin \mathrm{fn}(Q)) & \\
\mu X.P \equiv P\{\mu X.P/_X\} & &
\end{array}
$$

Fig. 7. Structural congruence.

A.1 Structural Congruence

The notion of bound and free identifiers is extended to cover the subject and objects of $\texttt{arrived}\, u$, $\texttt{arrived}\, k$, $\texttt{arrived}\, k\, h$, $\texttt{typecase}\, k\, \texttt{of}\, \{(x_i{:}T_i)\, P_i\}_{i\in I}$, $\overline{a}\,\langle s\rangle$, $a\,[\vec{s}]$, and $s\,[S, \texttt{i}{:}\vec{h}, \texttt{o}{:}\vec{h'}]$ in the expected way. We write $\mathrm{fn}(P)$ for the set of names that have a free occurrence in P. Then structural congruence is the smallest congruence on processes generated by the following rules in Figure 7.[3]

A.2 Reduction

Reduction Relation The binary single-step reduction relation, $\longrightarrow$ is the smallest relation on closed terms generated by the rules in Figure 3 together with those in Figure 8.

B Appendix to Section 2.3

B.1 Subtyping

If P has a session channel s typed by S, P can interact at *s at most* as S (e.g. if S has shape $\oplus\{l_1 : S_1, l_2 : S_2, l_3 : S_3\}$ then P may send l_1 or l_3, but not a label different from $\{l_1, l_2, l_3\}$). Hence, $S \leq S'$ means that a process with a session typed by S is more composable with a peer process than one by S'. Composability also characterises the subtyping on shared channel types. Formally the subtyping relation is defined for the set $\mathscr{T}$ of all closed and contractive types as follows: T is a subtype of T', written $T \leq T'$, if (T, T') is in the largest fixed point of the monotone function $\mathscr{F} : \mathscr{P}(\mathscr{T} \times \mathscr{T}) \to \mathscr{P}(\mathscr{T} \times \mathscr{T})$, such that $\mathscr{F}(\mathscr{R})$ for each $\mathscr{R} \subset \mathscr{T} \times \mathscr{T}$ is given as follows.

$$
\begin{array}{ll}
 & \{(\mathsf{bool}, \mathsf{bool}), (\mathsf{nat}, \mathsf{nat})\} \cup \{(\texttt{o}\langle S\rangle, \texttt{o}\langle S'\rangle) \mid (S, S'), (S', S) \in \mathscr{R}\} \\
\cup & \{(\texttt{i}\langle S\rangle, \texttt{i}\langle S'\rangle) \mid (S, S'), (S', S) \in \mathscr{R}\} \\
\cup & \{(\mu\, \mathsf{X}.U, U') \mid (U\{\mu\, \mathsf{X}.U/_{\mathsf{X}}\}, U') \in \mathscr{R}\} \cup \{(U, \mu\, \mathsf{X}.U') \mid (U, U'\{\mu\, \mathsf{X}.U'/_{\mathsf{X}}\}) \in \mathscr{R}\} \\
\cup & \{(!(T_1); S_1', !(T_2); S_2') \mid (T_2, T_1), (S_1', S_2') \in \mathscr{R}\} \cup \{(?(T_1); S_1', ?(T_2); S_2') \mid (T_1, T_2), (S_1', S_2') \in \mathscr{R}\} \\
\cup & \{(\oplus\{l_i : S_i\}_{i\in I}, \oplus\{l_j : S_j'\}_{j\in J}) \mid \forall i \in I \subseteq J.(S_i, S_i') \in \mathscr{R}\} \\
\cup & \{(\&\{l_i : S_i\}_{i\in I}, \&\{l_j : S_j'\}_{j\in J}) \mid \forall j \in J \subseteq I.(S_j, S_j') \in \mathscr{R}\} \\
\cup & \{(\mu\, \mathsf{X}.S, S') \mid (S\{\mu\, \mathsf{X}.S/_{\mathsf{X}}\}, S') \in \mathscr{R}\} \cup \{(S, \mu\, \mathsf{X}.S') \mid (S, S'\{\mu\, \mathsf{X}.S'/_{\mathsf{X}}\}) \in \mathscr{R}\} \\
\cup & \{(\{S_i\}_{i\in I}, \{S_j'\}_{j\in J}) \mid \neg(|I| = |J| = 1), \forall j \in J, \exists i \in I.(S_i, S_j') \in \mathscr{R}\}
\end{array}
$$

Line 1 is standard ($\langle S\rangle$ is invariant at S since it logically contains both S and $\overline{S}$). Line 2/6 are the standard rule for recursion. In Line 3, the linear output (resp. input) is contravariant

[3] For the recursion, it would be more natural to use a demand-driven input-guarded reduction for the LN-transformation. The choice does not affect the developments of this paper.

$$
\begin{array}{lrcll}
\text{[Request1]} & \overline{a}(x{:}S);P & \longrightarrow & (\nu\, s)\,(P\{\overline{s}/x\} \mid \overline{s}\,[S, \mathtt{i}{:}\varepsilon, \mathtt{o}{:}\varepsilon] \mid \overline{a}\,\langle s\rangle) & (s \notin \mathrm{fn}(P)) \\
\text{[Request2]} & a\,[\vec{s}] \mid \overline{a}\,\langle s\rangle & \longrightarrow & a\,[\vec{s}{\cdot}s] & \\
\text{[Accept]} & a(x{:}S).P \mid a\,[s{\cdot}\vec{s}] & \longrightarrow & P\{s/x\} \mid s\,[S, \mathtt{i}{:}\varepsilon, \mathtt{o}{:}\varepsilon] \mid a\,[\vec{s}] & \\
\text{[Send]} & s!\langle v\rangle;P \mid s\,[!(T);S, \mathtt{o}{:}\vec{h}] & \longrightarrow & P \mid s\,[S, \mathtt{o}{:}\vec{h}{\cdot}v] & \\
\text{[Receive]} & s?(x).P \mid s\,[?(T);S, \mathtt{i}{:}v{\cdot}\vec{h}] & \longrightarrow & P\{v/x\} \mid s\,[S, \mathtt{i}{:}\vec{h}] & \\
\text{[Sel], [Bra]} & s \triangleleft l_i;P \mid s\,[\oplus\{l_i{:}S_i\}_{i\in I}, \mathtt{o}{:}\vec{h}] & \longrightarrow & P \mid s\,[S_i, \mathtt{o}{:}\vec{h}{\cdot}l_i] & (i \in I) \\
& s \triangleright \{l_j{:}P_j\}_{j\in J} \mid s\,[\&\{l_i{:}S_i\}_{i\in I}, \mathtt{i}{:}l_i{\cdot}\vec{h}] & \longrightarrow & P_i \mid s\,[S_i, \mathtt{i}{:}\vec{h}] & (i \in I \subseteq J) \\
\text{[Comm]} & s\,[\mathtt{o}{:}v{\cdot}\vec{h}] \mid \overline{s}\,[\mathtt{i}{:}\vec{h}'] & \longrightarrow & s\,[\mathtt{o}{:}\vec{h}] \mid \overline{s}\,[\mathtt{i}{:}\vec{h}'{\cdot}v] & \\
\text{[Arriv-req]} & E[\mathtt{arrived}\,a] \mid a\,[\vec{s}] & \longrightarrow & E[\mathtt{b}] \mid a\,[\vec{s}] & (|\vec{s}| \geq 1) \searrow \mathtt{b} \\
\text{[Arriv-ses]} & E[\mathtt{arrived}\,s] \mid s\,[\mathtt{i}{:}\vec{h}] & \longrightarrow & E[\mathtt{b}] \mid s\,[\mathtt{i}{:}\vec{h}] & (|\vec{h}| \geq 1) \searrow \mathtt{b} \\
\text{[Arriv-msg]} & E[\mathtt{arrived}\,s\,h] \mid s\,[\mathtt{i}{:}\vec{h}] & \longrightarrow & E[\mathtt{b}] \mid s\,[\mathtt{i}{:}\vec{h}] & (\vec{h} = h{\cdot}\vec{h}') \searrow \mathtt{b} \\
\text{[Typecase]} & & & & \\
\multicolumn{5}{l}{\quad \mathtt{typecase}\,s\,\mathtt{of}\,\{(x_i{:}T_i)\,P_i\}_{i\in I} \mid s\,[S] \longrightarrow P_i\{s/x_i\} \mid s\,[S] \qquad \exists i \in I.(\forall j < i.T_j \not\leq S \wedge T_i \leq S)}
\end{array}
$$

$$
\text{[Unfold]}\ P \mid s\,[S\{\mu\mathsf{X}.S/\mathsf{X}\}, \mathtt{i}{:}\vec{h}_1, \mathtt{o}{:}\vec{h}'_1] \longrightarrow P' \mid s\,[S', \mathtt{i}{:}\vec{h}_2, \mathtt{o}{:}\vec{h}'_2] \implies P \mid s\,[\mu\mathsf{X}.S, \mathtt{i}{:}\vec{h}_1, \mathtt{o}{:}\vec{h}'_1] \longrightarrow P' \mid s\,[S', \mathtt{i}{:}\vec{h}_2, \mathtt{o}{:}\vec{h}'_2]
$$

$$
\begin{array}{lrcl}
\text{[Eval]} & e \longrightarrow e' & \implies & E[e] \longrightarrow E[e'] \\
\text{[Chan]} & P \longrightarrow P' & \implies & (\nu\, a{:}\langle S\rangle)\,P \longrightarrow (\nu\, a{:}\langle S\rangle)\,P' \\
\text{[Sess]} & P \longrightarrow P' & \implies & (\nu\, s)\,P \longrightarrow (\nu\, s)\,P' \\
\text{[If-true]} & & & \mathtt{if}\,\mathtt{tt}\,\mathtt{then}\,P\,\mathtt{else}\,Q \longrightarrow P \\
\text{[If-false]} & & & \mathtt{if}\,\mathtt{ff}\,\mathtt{then}\,P\,\mathtt{else}\,Q \longrightarrow Q \\
\text{[Par]} & P \longrightarrow P' & \implies & P \mid Q \longrightarrow P' \mid Q \\
\text{[Struct]} & P \equiv P' \longrightarrow Q' \equiv Q & \implies & P \longrightarrow Q
\end{array}
$$

Fig. 8. The reduction rules § 2.2.

(resp. covariant) on its carried types following [20]. In Line 4, the selection is co-variant since if a process may send more labels, it is less composable with its peer. Dually for branching in Line 5. Finally, the ordering of the set types says that if each element of the set type $\{S'_j\}_{j\in J}$ has its subtype in $\{S_i\}_{i\in I}$, the former is less composable by the latter. The condition $\neg(|I| = |J| = 1)$ avoids the case $\{S_i\}_{i\in I} = S, \{S'_j\}_{j\in J} = S'$, which makes the relation universal.

We now clarify the semantics of $\leq$ using *duality*. The dual of S, denoted $\overline{S}$, is defined in the standard way: $\overline{!(T);S} = ?(T);\overline{S}$, $\overline{?(T);S} = !(T);\overline{S}$, $\overline{\mu\mathsf{X}.S} = \mu\mathsf{X}.\overline{S}$, $\overline{\mathsf{X}} = \mathsf{X}$, $\overline{\oplus\{l_i{:}S_i\}_{i\in I}} = \&\{l_i{:}\overline{S_i}\}_{i\in I}$ $\overline{\&\{l_i{:}S_i\}_{i\in I}} = \oplus\{l_i{:}\overline{S_i}\}_{i\in I}$ and $\overline{\mathtt{end}} = \mathtt{end}$.

B.2 Types and Typing

The type syntax is defined in the main section. We use following extended types for typing queues:

$$
\begin{array}{llll}
\text{(Message)} & M ::= I \mid O & \text{(General)} & \mathsf{T} ::= M \mid M;S \\
\text{(OMsg)} & O ::= \emptyset \mid !(T) \mid \oplus l \mid O;O' & \text{(IMsg)} & I ::= \emptyset \mid ?(T) \mid \&l \mid I;I'
\end{array}
$$

Message types M represent either input I or output O in the input and output queues, respectively. Input messages $?(T)$ and $\&l$ type values and labels in the input queue respectively. Similarly for output messages $!(T)$ and $\oplus l$ for the output queue. $\emptyset$ is used to type empty buffers. A general type can be a session type a message type or a session type prefixed be a session type.

B.3 Typing System

We first define the $*$ commutative operator for typing parallel processes.

$$
\begin{array}{lll}
S*!(T);O = !(T);S*O & S_k * \oplus l_k;O = \oplus\{l_i : S_i\}_{i\in I} * O \ (l_k \in I) & \\
?(T);S*?(T);I = S*I & \&\{l_i : S_i\}_{i\in I} * \&l_k;I = S_k * I \quad (l_k \in I) & \emptyset * \mathsf{T} = \mathsf{T} \\
\Delta * \Delta' = \Delta \setminus \mathrm{dom}(\Delta') \cup \Delta' \setminus \mathrm{dom}(\Delta) \cup \{s : \mathsf{T} * \mathsf{T}' \mid \mathsf{T} \in \Delta, \mathsf{T}' \in \Delta'\} &&
\end{array}
$$

Figure 9 introduces the typing system for queues, which are with message types M. Note that (1) s is recorded to avoid a parallel composition of two queues in the same session name: and (2) OMsg is used to type the output queue, while IMsg is so to type the input queue.

Figure 10 gives the typing system for processes. The system is an extension of [2] with the two input and output entries.

$$
\Gamma \vdash s[\mathtt{i} : \varepsilon] \triangleright s : \emptyset, s \quad (\mathsf{InQ}) \qquad \Gamma \vdash s[\mathtt{o} : \varepsilon] \triangleright s : \emptyset, s \quad (\mathsf{OutQ})
$$

$$
\frac{\Gamma \vdash s[\mathtt{i} : h] \triangleright s : I, s \quad \Gamma \vdash v : T}{\Gamma \vdash s[\mathtt{i} : v \cdot h] \triangleright s : ?(T); I, s} \ (\mathsf{RcvQ}) \qquad \frac{\Gamma \vdash s[\mathtt{i} : h] \triangleright s : I, s}{\Gamma \vdash s[\mathtt{i} : l \cdot h] \triangleright s : \&l; I, s} \ (\mathsf{BraQ})
$$

$$
\frac{\Gamma \vdash s[\mathtt{o} : h] \triangleright s : O, s \quad \Gamma \vdash v : T}{\Gamma \vdash s[\mathtt{o} : v \cdot h] \triangleright s : !(T); O, s} \ (\mathsf{SndQ}) \qquad \frac{\Gamma \vdash s[\mathtt{o} : h] \triangleright s : O, s}{\Gamma \vdash s[\mathtt{o} : h \cdot l] \triangleright s : \oplus l; O, s} \ (\mathsf{SelQ})
$$

$$
\frac{\Gamma \vdash s[\mathtt{i} : h] \triangleright s : I, s}{\Gamma \vdash s[\mathtt{i} : s' \cdot h] \triangleright s : ?(S'); I \cdot s' : S', s} \ (\mathsf{InDelQ}) \qquad \frac{\Gamma \vdash s[\mathtt{o} : h] \triangleright s : O, s \cdot s' : S'}{\Gamma \vdash s[\mathtt{o} : s' \cdot h] \triangleright s : !(S'); O, s} \ (\mathsf{DelQ})
$$

Fig. 9. Typing system for queues

B.4 Proof of Subject Reduction Theorem (Proposition 2.1)

This appendix proves the subject reduction theorem of the asynchronous session types typing system.

First we note that:

Lemma B.1. *Let $P = Q \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon]$ and $\Gamma \vdash P \triangleright \Delta$ with $\mathtt{wc}(\Delta)$. Then if $P \twoheadrightarrow Q' \mid s[\mathtt{i} : \vec{h_1}, \mathtt{o} : \vec{h_2}]$ then $\vec{h_1} = \emptyset$ or $\vec{h_2} = \emptyset$.*

Theorem B.1 (Subject Reduction). (Proposition 2.1) *If $\Gamma \vdash P \triangleright \Delta$, Δ is well-configures and $P \twoheadrightarrow P'$ then $\Gamma \vdash P \triangleright \Delta'$ and $\Delta \twoheadrightarrow \Delta'$ and Δ' is well-configured.*

Proof. The proof is done by induction on the reduction relation.
The proof for subject congruence (i.e. the case for $P \equiv Q$) is the standard. Hence we prove the case for $P \longrightarrow Q$.
Case [Request1]. $\Gamma \vdash \overline{a}(x); P \triangleright \Delta \longrightarrow (\nu\ s)(P\{\overline{s}/x\} \mid \overline{a}\langle s\rangle \mid \overline{s}[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon]) \triangleright \Delta'$. By rule (Req), we have that $\Gamma \vdash P \triangleright \Delta \cdot \overline{x} : \overline{S}$. By rules (InQ, OutQ), we obtain that $\Gamma \vdash s[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon] \triangleright \emptyset$. Then by rule (Areq), we have $\Gamma \vdash \overline{a}(s); \triangleright s : S$. We now apply rule (Conc) to obtain $\Gamma \vdash P\{\overline{s}/x\} \mid \overline{a}\langle s\rangle \mid \overline{s}[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon] \triangleright \Delta \cdot s : S \cdot \overline{s} : \overline{S}$. Rule (Sres) gives us $\Gamma \vdash (\nu\ s)(P\{\overline{s}/x\} \mid \overline{a}\langle s\rangle \mid \overline{s}[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon]) \triangleright \Delta$, as required.
Case [Request2]. $\Gamma \vdash \overline{a}\langle s\rangle \mid a[\vec{s}] \triangleright \vec{s} : \vec{S} \cdot s : S \longrightarrow a[\vec{s} \cdot s] \triangleright \vec{s} : \vec{S} \cdot s : S$. We type the processes that compose the left hand side process using typing rules (Queue), (Areq). By rule (Conc) and the definition of $*$ we get the typing $\vec{s} : \vec{S} \cdot s : S$. The right hand side is typed using typing rule (Areq) to get the same result.
Case [Accept]. $\Gamma \vdash a(x).P \mid a[s \cdot \vec{s}] \triangleright \Delta \cdot \vec{s} : \vec{S} \cdot s : S \longrightarrow P\{s/x\} \mid a[\vec{s}] \triangleright \Delta \cdot \vec{s} : \vec{S} \cdot s : S$. For the left

$$\Gamma \cdot u : U \vdash u : U \quad \text{(Name)} \qquad \Gamma \vdash \texttt{tt}, \texttt{ff} : \mathsf{bool} \quad \text{(Bool)}$$

$$\frac{\Gamma \vdash e_i : \mathsf{bool}}{\Gamma \vdash e_1 \ \texttt{and} \ e_2 : \mathsf{bool}} \quad \text{(And)} \qquad \Gamma \cdot u : \mathtt{i}\langle S \rangle \vdash u : \mathtt{o}\langle S \rangle \quad \text{(Name')}$$

$$\frac{\Gamma \vdash u : \mathtt{i}\langle S \rangle}{\Gamma \vdash \texttt{arrive}\ a : \mathsf{bool}} \quad \text{(Areq)} \qquad \frac{\Gamma \vdash v : U}{\Gamma, \Delta \cdot s : ?(U); S \vdash \texttt{arrive}\ k\ v : \mathsf{bool}} \quad \text{(Amsg)}$$

$$\frac{\Gamma, \Delta \vdash \texttt{arrive}\ k\ v : \mathsf{bool}}{\Gamma \vdash \texttt{arrive}\ k\ : \mathsf{bool}} \quad \text{(Asess)}$$

$$\frac{\Gamma \vdash a : \mathtt{o}\langle S \rangle \quad \Gamma \vdash P \triangleright \Delta \cdot x : S}{\Gamma \vdash \overline{a}(x); P \triangleright \Delta} \quad \text{(Req)} \qquad \frac{\Gamma \vdash a : \mathtt{i}\langle S \rangle \quad \Gamma \vdash P \triangleright \Delta \cdot x : S}{\Gamma \vdash a(x).P \triangleright \Delta} \quad \text{(Acc)}$$

$$\frac{\Gamma \vdash v : U \quad U \neq \mathtt{i}\langle S \rangle \quad \Gamma \vdash P \triangleright \Delta \cdot k : S}{\Gamma \vdash k!\langle v \rangle; P \triangleright \Delta \cdot k : !(U); S} \quad \text{(Send)} \qquad \frac{\Gamma \cdot x : U \vdash P \triangleright \Delta \cdot k : S \quad U \neq \mathtt{i}\langle S \rangle}{\Gamma \vdash k?(x); P \triangleright \Delta \cdot k : ?(U); S} \quad \text{(Recv)}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot k : S}{\Gamma \vdash k!\langle k' \rangle; P \triangleright \Delta \cdot k : !(S'); S \cdot k' : S'} \quad \text{(Deleg)} \qquad \frac{\Gamma \vdash P \triangleright \Delta \cdot k : S \cdot k' : S'}{\Gamma \vdash k?(x); P \triangleright \Delta \cdot k : ?(S'); S} \quad \text{(SRecv)}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot k : S}{\Gamma \vdash k \oplus l; P \triangleright \Delta \cdot k : \oplus \{l_i : S_i\}_{i \in I}} \quad \text{(Sel)} \qquad \frac{\Gamma \vdash P_i \triangleright \Delta \cdot k : S_i \quad \forall\, i \in I}{\Gamma \vdash k \& \{l_i : P_i\}_{i \in I} \triangleright \Delta \cdot k : \& \{l_i : S_i\}_{i \in I}} \quad \text{(Bra)}$$

$$\frac{\begin{array}{c}\Gamma \vdash P_i \triangleright \Delta_i \ (i = 1,2) \\ \text{if } s \in \Delta_i \text{ then } s \notin \Delta_j (i \neq j)\end{array}}{\Gamma \vdash P_1 \mid P_2 \triangleright \Delta_1 * \Delta_2} \quad \text{(Conc)} \qquad \frac{\Gamma, \Delta \vdash e : \mathsf{bool} \quad \Gamma \vdash P \triangleright \Delta \quad \Gamma \vdash Q \triangleright \Delta}{\Gamma \vdash \texttt{if}\ e\ \texttt{then}\ P\ \texttt{else}\ Q \triangleright \Delta} \quad \text{(If)}$$

$$\frac{\Delta \ \texttt{end only}}{\Gamma \vdash \mathbf{0} \triangleright \Delta} \quad \text{(Inact)} \qquad \frac{\Gamma \cdot a : U \vdash P \triangleright \Delta \cdot a}{\Gamma \vdash (\nu\ a)P \triangleright \Delta} \quad \text{(NRes)}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot s : S \cdot \overline{s} : \overline{S}, s}{\Gamma \vdash (\nu\ s)P \triangleright \Delta} \quad \text{(SRes)} \qquad \frac{\Delta \ \texttt{end only}}{\Gamma \vdash a[\varepsilon] \triangleright \Delta \cdot a} \quad \text{(EQueue)}$$

$$\frac{\Gamma \vdash a[\vec{h}] \triangleright \Delta}{\Gamma \vdash a[\vec{h} \cdot s] \triangleright \Delta \cdot s : S} \quad \text{(Queue)} \qquad \Gamma \cdot X : \Delta \vdash X \triangleright \Delta \quad \text{(Var)}$$

$$\frac{\Gamma \cdot X : \Delta \vdash P \triangleright \Delta}{\Gamma \vdash \mu X.P \triangleright \Delta} \quad \text{(Rec)} \qquad \Gamma \vdash \overline{a}\langle s \rangle \triangleright s : S \quad \text{(AReq)}$$

$$\frac{\forall i \in I \quad \Gamma \vdash P_i \triangleright \Delta \cdot x_i : T_i \quad \cup_{i \in I} T_i \leq T}{\Gamma \vdash \texttt{typecase}\ k\ \texttt{of}\ \{(x_i : T_i)\, P_i\}_{i \in I} \triangleright \Delta \cdot k : T} \quad \text{(Typecase)}$$

We write $\Gamma \vdash P \triangleright \Delta$ as $\Gamma \vdash_{\emptyset} P \triangleright \Delta$.

Fig. 10. Typing System

hand side, we use rules (Queue), (Acc) and (Conc), to get the typing result. From rule (Acc) we have that $\Gamma \vdash P\{s/x\} \triangleright \Delta$. From here is easy to find the same typing for the right hand side.

Case [Send] (Value)**.** $\Gamma \vdash s!\langle v \rangle; P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h}] \triangleright \Delta \cdot s : S, s \longrightarrow P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : v \cdot \vec{h}] \triangleright \Delta \cdot s : S, s$, where $\Gamma \vdash \vec{h} : \vec{T}, \Gamma \vdash v : T$. For the left hand side we type $\Gamma \vdash s!\langle v \rangle; P \triangleright \Delta \cdot s : !(T); S'$ and $\Gamma \vdash s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h}] \triangleright s : O, s$. Using (Conc) we get $!(T); S' * O = S$. Now if we type the right hand side we get $\Gamma \vdash s!\langle v \rangle; P \triangleright \Delta \cdot s : S'$ and $\Gamma \vdash s[\mathtt{i} : \varepsilon, \mathtt{o} : v \cdot \vec{h}] \triangleright s : !(T); O, s$. We compose to get $S' * !(T); O = !(T); S' * O = S$.

Case [Receive] (Value)**.** $\Gamma \vdash s?(x); P \mid s[\mathtt{i} : v \cdot \vec{h}, \mathtt{o} : \varepsilon] \triangleright \Delta \cdot s : S, s \longrightarrow P\{v/x\} \mid s[\mathtt{i} : \vec{s}, \mathtt{o} :$

$\varepsilon] \triangleright \Delta \cdot s : S, s$. For the left hand side we have $\Gamma \vdash s?(x); P \triangleright \Delta \cdot s :?(T); S'$ and $\Gamma \vdash s[\mathtt{i} : v \cdot \vec{h}, \mathtt{o} : \varepsilon] \triangleright s :?(T); I, s$. We compose and get $?(T); S' * ?(T); I = S' * I = S$. For the right hand side we have $\Gamma \vdash P \triangleright \Delta \cdot s : S'$ and $\Gamma \vdash s[\mathtt{i} : \vec{h}, \mathtt{o} : \varepsilon] \triangleright s : I, s$. By composition we get $S' * I = S$.
Case [Receive] (Delegation)**.** $\Gamma \vdash s?(x); P \mid s[\mathtt{i} : s' \cdot \vec{h}] \triangleright \Delta \cdot s : S \cdot s' : S', s \longrightarrow P\{s'/x\} \mid s[\mathtt{i} : \vec{h}] \triangleright \Delta \cdot s : S \cdot s' : S', s$. We have $\Gamma \vdash s?(x); P \triangleright \Delta \cdot s :?(S'); S'', \Gamma \vdash s[\mathtt{i} : s' \cdot \vec{h}] \triangleright s :?(S'); I \cdot s' : S', s$ and $\Delta \cdot s :?(S'); S'' * s :?(S'); I \cdot s' : S', s = \Delta \cdot s :?(S'); S \cdot s' : S', s$. For the right hand side we have $\Gamma \vdash P\{s'/x\} \triangleright \Delta \cdot s : S'' \cdot s' : S', \Gamma \vdash s[\mathtt{i} : \vec{h}] \triangleright s : I, s$ and $\Delta \cdot s : S'' \cdot s' : S' * s : I, s = \Delta \cdot s :?(S'); S \cdot s' : S', s$.
Case [Send] (Delegation)**.** Similar to the above case.
Case [Sel]**.** $\Gamma \vdash s \oplus v; P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h}] \triangleright \Delta \cdot s : S, s \longrightarrow P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : l \cdot \vec{h}] \triangleright \Delta \cdot s : S, s$. The proof is similar to [Send] case.
Case [Bra]**.** $\Gamma \vdash s\&\{l_i : P_i\}_{i \in I} \mid s[\mathtt{i} : l_k \cdot \vec{h}, \mathtt{o} : \varepsilon] \triangleright \Delta \cdot s : S', s \longrightarrow P_k \mid s[\mathtt{i} : \vec{s}, \mathtt{o} : \varepsilon] \triangleright \Delta \cdot s : S', s$ where $S' = S_k * I$ and $\Gamma \vdash s[\mathtt{i} : \vec{s}, \mathtt{o} : \varepsilon] \triangleright s : I, s$. The proof is similar to the (Receive) case.
Case [Comm]**.** $\Gamma \vdash P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h} \cdot v] \mid \overline{s}[\mathtt{i} : \vec{h'}, \mathtt{o} : \varepsilon] \triangleright \Delta_1 \longrightarrow P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h}] \mid \overline{s}[\mathtt{i} : \vec{h'} \cdot v, \mathtt{o} : \varepsilon] \triangleright \Delta_1$. $\Gamma \vdash P \triangleright \Delta \cdot s : S_1 \cdot \overline{s} : S_2$ with $\Gamma \vdash P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h} \cdot v] \mid \overline{s}[\mathtt{i} : \vec{h'}, \mathtt{o} : \varepsilon] \triangleright \Delta \cdot s :!(T); S \cdot \overline{s} :?(T); \overline{S}$ from the induction hypothesis. If we type the right hand side we have that $\Gamma \vdash P \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \vec{h}] \mid \overline{s}[\mathtt{i} : \vec{h'} \cdot v, \mathtt{o} : \varepsilon] \triangleright \Delta \cdot s : S \cdot \overline{s} : \overline{S}$ as required.

C Appendix for Section 3

Transitions: We write $\Longrightarrow$ for the reflexive and transitive closure of $\xrightarrow{\tau}$, $\overset{\ell}{\Longrightarrow}$ for the composition $\Longrightarrow\xrightarrow{\ell}\Longrightarrow$ and $\overset{\hat{\ell}}{\Longrightarrow}$ for $\Longrightarrow$ if $\ell = \tau$ and $\overset{\ell}{\Longrightarrow}$ otherwise. Furthermore we write $\xrightarrow{\hat{\ell}}$ for $\longrightarrow$ if $\ell = \tau$ and $\xrightarrow{\ell}$ otherwise.
Subject and Object: We define the subject ($\mathsf{subj}(\ell)$) and object ($\mathsf{obj}(\ell)$) for actions ℓ. For session transitions, we have $\mathsf{subj}(a\langle s\rangle) = \mathsf{subj}(\overline{a}\langle s\rangle) = \mathsf{subj}(\overline{a}(s)) = \{a\}$ and $\mathsf{obj}(a\langle s\rangle) = \mathsf{obj}(\overline{a}\langle s\rangle) = \mathsf{obj}(\overline{a}(s)) = \{s\}$. For session transitions, we have $\mathsf{subj}(s!\langle v\rangle) = \mathsf{subj}(s?\langle x\rangle) = \mathsf{subj}(s!(a)) = \mathsf{subj}(s \oplus l) = \mathsf{subj}(s\&l) = \{a\}$ and $\mathsf{obj}(s!\langle v\rangle) = \{v\}$, $\mathsf{obj}(s?\langle x\rangle) = \{x\}$, $\mathsf{subj}(s!(a)) = \{a\}$ $\mathsf{subj}(s \oplus l) = \mathsf{subj}(s\&l) = \{l\}$.
Free and Bound Names: Free and bound names follow the definition, $\mathsf{fn}(a\langle s\rangle) = \mathsf{fn}(\overline{a}\langle s\rangle) = \{a, s\}$, $\mathsf{fn}(\overline{a}(s)) = \{a\}$, $\mathsf{bn}(a\langle s\rangle) = \mathsf{bn}(\overline{a}\langle s\rangle) = \emptyset$, $\mathsf{bn}(\overline{a}(s)) = \{s\}$ for shared name actions. For session name actions, we have $\mathsf{fn}(s!\langle v\rangle) = \mathsf{fn}(s?\langle x\rangle) = \mathsf{fn}(s!(a)) = \mathsf{fn}(s \oplus l) = \mathsf{fn}(s\&l) = \{s\}$, where v is not a name, $\mathsf{fn}(s!\langle a\rangle) = \{s, a\}$, $\mathsf{fn}(s!\langle s'\rangle) = \{s, s'\}$ and $\mathsf{bn}(s!\langle v\rangle) = \mathsf{fn}(s?\langle x\rangle) = \mathsf{subj}(s!\langle a\rangle) = \mathsf{bn}(s \oplus l) = \mathsf{bn}(s\&l) = \emptyset$, $\mathsf{bn}(s!(a)) = \{a\}$, $\mathsf{bn}(s!(s')) = \{s'\}$.
The names of an action ($\mathsf{n}(\ell)$) is the union of $\mathsf{fn}(\ell)$ and $\mathsf{bn}(\ell)$, $\mathsf{n}(\ell) = \mathsf{fn}(\ell) \cup \mathsf{bn}(\ell)$.

Definition C.1 (context). *A context is defined as*

$$\begin{array}{l} C ::= - \mid C \mid C' \mid (\nu\ n)C \mid \mathtt{if}\ e\ \mathtt{then}\ C\ \mathtt{else}\ C' \mid \mathbf{0} \mid \mathtt{X} \mid \mu X.C \mid \\ \quad s!\langle v\rangle; C \mid s?(x); C \mid s \oplus l; C \mid s\&\{l_i : C_i\}_{i \in I} \mid \overline{a}(x); C \mid a(x).C \mid \\ \quad \overline{a}\langle s\rangle \mid a[\vec{s}] \mid s[\mathtt{i} : \vec{h}, \mathtt{o} : \vec{h}] \end{array}$$

Expression $C[P]$ substitutes process P in each hole $(-)$ of the context C definition.

C.1 Proof for Theorem 3.3

Theorem (Coincidence) $\approx$ and $\cong$ coincide.

The above theorem requires to show the equality into two directions.

Lemma C.1 (Soundness). $\approx$ *implies* $\cong$

Proof. Reduction closeness and barb observation properties are easy to be verified. The only remaining property is showing that $\approx$ is a congruence.

Output, restriction, if construct and recursion congruence are easy to be verified. Input congruence is is similar to output congruence, since we are dealing with programs, which are processes without free variables We give the result for Parallel congruence.

Parallel Congruence
Assume relation

$$\mathscr{S} = \{((\nu\ \vec{a},\vec{s})(P \mid R),(\nu\ \vec{a},\vec{s})(Q \mid R)) \mid \tag{1}$$
$$P \approx Q, \forall R \cdot P \mid R, Q \mid R \text{ are typable}, \forall \vec{a},\vec{s}\} \tag{2}$$

We show that $\mathscr{S}$ is a typed relation.
Since $P \approx Q$ we have that $\Gamma \vdash P \triangleright \Delta$ and $\Gamma \vdash Q \triangleright \Delta'$ with $\Delta \bowtie \Delta'$. Since P, Q are localised and R is localised and $P \mid R, Q \mid R$ are typable then $\mathrm{dom}\,\Delta \cap \mathrm{dom}\,\Delta' = \emptyset$. Using Conc and the $*$ definition we get the result.

We show that S is a bisimulation. There are three cases:

Case (1) Suppose $\Gamma \vdash P \mid R \triangleright \Delta_1 \xrightarrow{\ell} P' \mid R \triangleright \Delta_1'$. Then $\Gamma \vdash P \triangleright \Delta_P \xrightarrow{\ell} P' \triangleright \Delta_P'$.
By the definition of $\mathscr{S}$, we have that $\Gamma \vdash Q \triangleright \Delta_Q \stackrel{\ell}{\Longrightarrow} Q' \triangleright \Delta_Q'$.
So have that $\Gamma \vdash Q \mid R \triangleright \Delta_2 \stackrel{\ell}{\Longrightarrow} Q' \mid R \triangleright \Delta_2'$.
Case (2) Suppose $\Gamma \vdash P \mid R \triangleright \Delta_1 \xrightarrow{\ell} P \mid R' \triangleright \Delta_1'$. Then $\Gamma \vdash R \triangleright \Delta_R \xrightarrow{\ell} R' \triangleright \Delta_R'$.
By the above, we have that $\Gamma \vdash Q \mid R \triangleright \Delta_2 \xrightarrow{\ell} Q \mid R' \triangleright \Delta_2'$. It remains to show that $\Delta_1' \bowtie \Delta_2'$. From here, we conclude $P \mid R \approx Q \mid R$ as required.

Case (3) Suppose $\Gamma \vdash P \mid R \triangleright \Delta_1 \longrightarrow (\nu\ \vec{a},\vec{s})(P' \mid R') \triangleright \Delta_1'$. Then we have

$$\Gamma \vdash P \triangleright \Delta_P \xrightarrow{\ell} P' \triangleright \Delta_P' \tag{3}$$
$$\tag{4}$$

By the definition of S, we have:

$$\Gamma \vdash Q \triangleright \Delta_Q \Longrightarrow \xrightarrow{\ell} \Longrightarrow Q' \triangleright \Delta_Q' \tag{5}$$

By (5), we have that $\Gamma \vdash Q \mid R \triangleright \Delta_2 \Longrightarrow (\nu\ \vec{a},\vec{s})(Q' \mid R') \triangleright \Delta_2'$. Then it remains to show that $\Delta_1' \bowtie \Delta_2'$.
The last result gives us that $P \mid R \approx Q \mid R$ as required. □

The proof for the completeness direction follows the technique shown in [6]. However we need to adapt it to session and buffers.

Definition C.2 (definability). *An external action* ℓ *is* definable *if for for a set of names* N*, action* $succ, \notin N$ *there is a* testing process $T\langle N, succ, \ell\rangle$ *with the property that for every process* P *and* $\mathsf{fn}(P) \subseteq N$

- $\Gamma \vdash P \triangleright \Delta_1 \xrightarrow{\ell} P' \triangleright \Delta_1'$ *implies that* $\Gamma \vdash T\langle N, succ, \ell\rangle \mid P \triangleright \Delta \twoheadrightarrow (\nu\ \mathsf{bn}(\ell), b)(succ[\mathsf{o} : \mathsf{bn}(\ell)] \mid R \mid P') \triangleright \Delta'$.

– $\Gamma \vdash T\langle N, succ, \ell\rangle \mid P \triangleright \Delta \twoheadrightarrow Q \triangleright \Delta'$, *where* $Q \Downarrow_{succ}$ *implies that* $Q = (\nu\ \mathsf{bn}(\ell), b)(succ[\mathtt{o} : \mathsf{bn}(\ell)] \mid R \mid P')$ *where* $\Gamma \vdash P \triangleright \Delta_1 \stackrel{\ell}{\Longrightarrow} P' \triangleright \Delta_1'$.

$R = b(x).R'$ or $R = \mathbf{0}$. Note that $b(x).R$ is used to keep the composition $P \mid T\langle N, \mathrm{succ}, \ell\rangle$ typable. Also $R \stackrel{l}{\not\longrightarrow}$ either due to the restriction of b, or because $R = \mathbf{0}$.

Lemma C.2. *Every external action is definable.*

Proof. The input action cases are straightforward:

1. If $\Gamma \vdash P \triangleright \Delta \stackrel{a(s)}{\longrightarrow} P' \triangleright \Delta'$ then $T\langle \emptyset, \mathrm{succ}, a(s)\rangle = \overline{a}(x); R \mid \mathrm{succ}[\mathtt{o} : \mathtt{tt}]$.
2. If $\Gamma \vdash P \triangleright \Delta \stackrel{s?\langle v\rangle}{\longrightarrow} P' \triangleright \Delta'$ then $T\langle \emptyset, \mathrm{succ}, s?\langle v\rangle\rangle = (\nu\ b)(s!\langle v\rangle; b(x).R) \mid \mathrm{succ}[\mathtt{o} : \mathtt{tt}]$.
3. If $\Gamma \vdash P \triangleright \Delta \stackrel{s\&l}{\longrightarrow} P' \triangleright \Delta'$ then $T\langle \emptyset, \mathrm{succ}, s\&l\rangle = (\nu\ b)(s \oplus l; b(x).R) \mid \mathrm{succ}[\mathtt{o} : \mathtt{tt}]$.

The requirements of Definition C.2 can be verified with simple transitions.

Output actions cases:

1. If $\Gamma \vdash P \triangleright \Delta \stackrel{\overline{a}\langle s\rangle}{\longrightarrow} P' \triangleright \Delta'$ then we have,

$$\begin{aligned} T\langle \{s\}, \mathrm{succ}, a\langle s\rangle\rangle = (\nu\ b)(a(x).&\\ &(\mathtt{if}\ x = s\ \mathtt{then}\ \mathrm{succ}!\langle x\rangle; R\\ &\mathtt{else}\ b(x).\mathrm{succ}!\langle x\rangle; R)) \mid\\ &\mathrm{succ}[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon] \mid a[\varepsilon] \end{aligned}$$

2. If $\Gamma \vdash P \triangleright \Delta \stackrel{s!\langle b\rangle}{\longrightarrow} P' \triangleright \Delta'$ then we have that

$$\begin{aligned} T\langle \{b\}, \mathrm{succ}, s!\langle b\rangle\rangle = (\nu\ b)(s?(x);&\\ &(\mathtt{if}\ x = b\ \mathtt{then}\ \mathrm{succ}!\langle x\rangle; b(x).R\\ &\mathtt{else}\ b(x).(\mathrm{succ}!\langle x\rangle; R)) \mid\\ &\mathrm{succ}[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon] \end{aligned}$$

3. If $\Gamma \vdash P \triangleright \Delta \stackrel{s!(b)}{\longrightarrow} P' \triangleright \Delta'$ then we have that

$$\begin{aligned} T\langle \{b\}, \mathrm{succ}, s!(b)\rangle = (\nu\ b)(s?(x);&\\ &(\mathtt{if}\ x = b\ \mathtt{then}\ \mathrm{succ}!\langle x\rangle; b(x).R\\ &\mathtt{else}\ b(x).(\mathrm{succ}!\langle x\rangle; R)) \mid\\ &\mathrm{succ}[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon] \end{aligned}$$

4. If $\Gamma \vdash P \triangleright \Delta \stackrel{s\oplus l_k}{\longrightarrow} P' \triangleright \Delta'$ then we have that:

$$T\langle \emptyset, \mathrm{succ}, s \oplus l_k\rangle = (\nu\ b)(s\&\{l_k : \mathrm{succ}!\langle \mathtt{tt}\rangle; R, l_i : b(x).R\}_{i\in I}), 1 \le i \le n$$

Again the requirements of Definition C.2 can be verified by simple transitions for each case.

Lemma C.3. *If succ is fresh, $b \in \vec{a} \cdot \vec{s}$ and*

$$\Gamma \vdash (\nu \vec{a},\vec{s},b)(P \mid succ[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta_1 \cong (\nu \vec{a},\vec{s},b)(Q \mid succ[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta_2 \quad (6)$$

then

$$\Gamma \vdash P \triangleright \Delta_P \cong Q \triangleright \Delta_Q \quad (7)$$

Proof. Let relation

$$\begin{aligned}\mathscr{S} = \{(\Gamma \vdash P \triangleright \Delta_P, \Gamma \vdash Q \triangleright \Delta_Q) \mid \\ \Gamma \vdash (\nu \vec{a},\vec{s},b)(P \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta_1 \\ \cong (\nu \vec{a},\vec{s},b)(Q \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta_2, \quad \text{succ is fresh}\}\end{aligned}$$

We will show that the contextual properties hold in $\mathscr{S}$.

Typing: It should hold that $\mathscr{S}$ is a typed relation. From $\mathscr{S}$ definition we have that $\Gamma \vdash (\nu\, \vec{a},\vec{s},b)(P \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta \approx (\nu\, \vec{a},\vec{s},b)(Q \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta', \Delta \bowtie \Delta'$. From here, by using typing rules (Nres), (Sres), (Conc), we get the required result.

Reduction Closedness: $\mathscr{S}$ is reduction closed by the freshness of succ. We cannot observed a reduction on succ or on $b(x).R$, so we conclude that if $\Gamma \vdash (\nu\, \vec{a},\vec{s},b)(P \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta \longrightarrow (\nu\, \vec{a},\vec{s},b)(P' \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta'$ implies $\Gamma \vdash (\nu\, \vec{a},\vec{s},b)(Q \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta \twoheadrightarrow (\nu\, \vec{a},\vec{s})(Q' \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \triangleright \Delta'$ then $\Gamma \vdash P \triangleright \Delta_1 \longrightarrow P' \triangleright \Delta_P$ implies $\Gamma \vdash Q \triangleright \Delta_1 \twoheadrightarrow Q' \triangleright \Delta_Q$

Preserve Observation: We do a case analysis on the cases where $P \downarrow_m$.

If $P \downarrow_m$, $m \notin \vec{a} \cdot \vec{s}$ and $(\nu\, \vec{a},\vec{s},b)(P \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \downarrow_m$ then $(\nu\, \vec{a},\vec{s},b)(Q \mid \mathrm{succ}[\mathtt{o} : a] \mid b(x).R) \Downarrow_m$. From the definition of $\mathscr{S}$ and the freshness of succ, we conclude $Q \Downarrow_m$.

If $P \downarrow_m$, $m \notin \vec{a} \cdot \vec{s}$ and $(\nu\, \vec{a},\vec{s},b)(P \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \not\downarrow_m$ then by the environment typing transition we have that m is a session occurring free in $\mathrm{succ}[\mathtt{o} : a'] \mid b(x).R$, and also $(\nu\, \vec{a},\vec{s},b)(Q \mid \mathrm{succ}[\mathtt{o} : a] \mid b(x).R) \not\Downarrow_m$. The case where $Q \not\Downarrow_m$ does not hold, because it would be possible to have $(\nu\, \vec{a},\vec{s},b)(Q \mid \mathrm{succ}[\mathtt{o} : a] \mid b(x).R) \mid Q'$ with Q' having as a free name session m and have a typable process. But composition $(\nu\, \vec{a},\vec{s},b)(P \mid \mathrm{succ}[\mathtt{o} : a'] \mid b(x).R) \mid Q'$ is untypable because $P \downarrow_m$, thus breaking reduction congruence. This results to the conclusion that $Q \downarrow_m$.

Context Property: The interesting case is the Parallel composition. We will show that if $\Gamma \vdash P \triangleright \Delta_P \mathscr{S} \Gamma \vdash Q \triangleright \Delta_Q$. Then for arbitrary process R we have that $\Gamma \vdash P \mid P_1 \triangleright \Delta'_P \mathscr{S} Q \mid P_1 \Delta'_Q$.

To show this, it is enough to show that $\Gamma \vdash (\nu\, \vec{a},\vec{s},b)(P \mid P_1 \mid \mathrm{succ}'[\mathtt{o} : a'] \mid R) \triangleright \Delta''_P \cong (\nu\, \vec{a},\vec{s},b)(Q \mid P_1 \mid \mathrm{succ}'[\mathtt{o} : a'] \mid R) \triangleright \Delta''_Q$, considering that succ may occur in P_1 and succ$'$ is fresh.

To prove this assume the process $\mathtt{tt}\langle\emptyset, \mathrm{succ}', \ell\rangle = \mathrm{succ}?(x); (\mathrm{succ}'!\langle x\rangle; \mathbf{0} \mid P'_1) \mid \mathrm{succ}'[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon]$, where $P_1 = P_1\{a'/x\}$.

From the contextual property of the theorem assumption and simple reductions, we have that:
$\Gamma \vdash (\nu\ \vec{a},\vec{s},b)(P \mid P_1 \mid \mathrm{succ}'[\mathsf{o} : a'] \mid R) \triangleright \Delta_1 \cong \Gamma \vdash (\nu\ \vec{a},\vec{s},b)(Q \mid P_1 \mid \mathrm{succ}'[\mathsf{o} : a'] \mid R) \triangleright \Delta_1'$.
We need to verify that
$\Gamma \vdash (\nu\ \vec{a},\vec{s},b)(P \mid P_1 \mid \mathrm{succ}'[\mathsf{o} : a'] \mid R) \triangleright \Delta_1 \approx (\nu\ \vec{a},\vec{s},b)(P \mid\ \mid P_1 \mid \mathrm{succ}'[\mathsf{o} : a'] \mid R) \triangleright \Delta_1'$,
which is simple because $R \approx \mathbf{0}$. By using lemma C.1 we get the result.

□

We are know ready to prove the completeness direction.

Lemma C.4 (Completness). $\cong$ *implies* $\approx$

Proof. For the proof we show that if

$$\Gamma \vdash P \triangleright \Delta_P \ \cong\ Q \triangleright \Delta_Q \text{ and} \tag{8}$$

$$\Gamma \vdash P \triangleright \Delta_P \xrightarrow{\ell} P' \triangleright \Delta_P' \tag{9}$$

then $\Gamma \vdash Q \triangleright \Delta_Q \stackrel{\ell}{\Longrightarrow} Q' \triangleright \Delta_Q'$ and $\Gamma \vdash P' \triangleright \Delta_P' \cong Q' \triangleright \Delta_Q'$

Suppose (8) and (9). Then there are two cases.
If $\ell = \tau$ then by reduction closeness of $\cong$ the result follows.
In the case where ℓ is an external action we can do a definability test for P by choosing the appropriate test $T\langle N, \mathrm{succ}, l\rangle$.
Because $\cong$ is context preserving we have that $\Gamma \vdash P \mid T\langle N, \mathrm{succ}, l\rangle \triangleright \Delta_{PT} \cong Q \mid T\langle N, \mathrm{succ}, l\rangle \triangleright \Delta_{QT}$. By Lemma C.2 we have that $\Gamma \vdash P \mid T\langle N, \mathrm{succ}, l\rangle \triangleright \Delta_{PT} \Longrightarrow (\nu\ \mathsf{bn}(\ell))(succ[\mathsf{o} : \mathsf{bn}(\ell)]|P') \triangleright \Delta$ so by the definition of $\cong$ (Definition 3.1), we have that $\Gamma \vdash T\langle N, \mathrm{succ}, l\rangle \mid Q \triangleright \Delta_{QT} \Longrightarrow R \triangleright \Delta'$. According to the second part of the Definition C.2, we can write:

$$\Gamma \vdash \Gamma \triangleright Q' = \Gamma \vdash (\nu\ \mathsf{bn}(\ell))(\mathrm{succ}[\mathsf{o} : \mathsf{bn}(\ell)] \mid b(x).\ \longrightarrow\ \mid Q'') \triangleright \Delta'' \tag{10}$$

$$\Gamma \vdash Q \triangleright \Delta_Q \stackrel{\ell}{\Longrightarrow} Q' \triangleright \Delta_Q' \tag{11}$$

Now we can derive $\Gamma \vdash \Gamma \triangleright (\nu\ \mathsf{bn}(\ell), b)(\mathrm{succ}[\mathsf{o} : \mathsf{bn}(\ell)] \mid b(x).\ \longrightarrow\ \mid P') \cong \Gamma \vdash (\nu\ \mathsf{bn}(\ell), b)(\mathrm{succ}[\mathsf{o} : \mathsf{bn}(\ell)] \mid b(x).\ \longrightarrow\ \mid Q') \triangleright \Delta''$. By Lemma C.3 we conclude that:

$$\Gamma \vdash P' \triangleright \Delta_P' \cong Q' \triangleright \Delta_Q' \tag{12}$$

$$\tag{13}$$

We began with the assumption that $\Gamma \vdash P \triangleright \Delta_P \cong \Gamma \vdash Q \triangleright \Delta_Q$ and we concluded to (11), (12). Thus $\cong$ implies $\approx$.

□

C.2 Bisimulation Properties

Proof for Lemma 3.4 We define input and output actions.

Definition (Input/Output Actions).

1. ℓ is an input action if $\ell \in \{a\langle s\rangle, a\langle s\rangle, s?\langle v\rangle, s?(v), s\&l_i\}$
2. ℓ is an output action if $\ell \in \{\overline{a}\langle s\rangle, \overline{a}(s), s!\langle v\rangle, s!(v), s \oplus l_i\}$

Lemma (Input and Output Asynchrony).

Suppose $\Gamma \vdash P \triangleright \Delta \stackrel{\ell}{\Longrightarrow} P' \triangleright \Delta'$.

1. *(input advance)* If ℓ is an input action, then $\Gamma \vdash P \triangleright \Delta \stackrel{\ell}{\longrightarrow\!\!\Longrightarrow} P' \triangleright \Delta'$.
2. *(output delay)* If ℓ is an output action, then $\Gamma \vdash P \triangleright \Delta \Longrightarrow\!\!\stackrel{\ell}{\longrightarrow} P' \triangleright \Delta'$.

Proof. For the first part of Lemma 3.4, there are two cases.

Case (1) P has the form $P = R \mid a[\vec{s}]$. $\Gamma \vdash R \mid a[\vec{s}] \triangleright \Delta \stackrel{a?\langle s\rangle}{\Longrightarrow} R' \mid a[\vec{s'} \cdot s] \triangleright \Delta'$ with $\vec{s} = s_\delta \cdot \vec{s'}$. Now we can observe $\Gamma \vdash R \mid a[\vec{s}] \triangleright \Delta \stackrel{a?\langle s\rangle}{\longrightarrow} R \mid a[\vec{s} \cdot s] \triangleright \Delta'' \Longrightarrow R' \mid a[\vec{s'} \cdot s] \triangleright \Delta'$ as required.
Case (2) Input communication takes place on a session channel. It is similar using a session queue.
For the second part of Lemma 3.4, there are two cases.

The first case is the action happening on a shared channel. Then $\Gamma \vdash P \triangleright \Delta \Longrightarrow P'' \mid \overline{a}\langle s\rangle \stackrel{a!\langle s\rangle}{\longrightarrow} P'' \Longrightarrow P' \triangleright \Delta'$.
From this we can always conclude that $\Gamma \vdash P \triangleright \Delta \Longrightarrow P' \mid \overline{a}\langle s\rangle \stackrel{a!\langle s\rangle}{\longrightarrow} P' \triangleright \Delta'$.
It is obvious that $\Delta = \Delta'$.
For the second case, we have the action to be observed on a session channel. $\Gamma \vdash P \mid s[\mathtt{o} : v \cdot \vec{h}] \triangleright \Delta \stackrel{s!\langle v\rangle}{\Longrightarrow} P' \mid s[\mathtt{o} : \vec{h'}] \triangleright \Delta'$, where $\vec{h'} = \vec{h} \cdot \vec{h_\delta}$.
Thus we can see that $\Gamma \vdash P \mid s[\mathtt{o} : v \cdot \vec{h}] \triangleright \Delta \Longrightarrow P' \mid s[\mathtt{o} : v \cdot \vec{h'}] \stackrel{s!\langle v\rangle}{\longrightarrow} P' \mid s[\mathtt{o} : \vec{h'}] \triangleright \Delta'$, where $v \cdot \vec{h'} = v \cdot \vec{h} \cdot \vec{h_\delta}$, which concludes the same result as above, with $\Delta = \Delta'$. □

Proof for Lemma 3.8

Lemma. Let P be session determinate and $\Gamma \vdash P \triangleright \Delta \Longrightarrow Q \triangleright \Delta'$. Then $P \approx Q$.

Proof. The proof considers induction on the length of $\Longrightarrow_s$ transition. The basic step is trivial. For the induction step step we do a case analysis on $\longrightarrow_s$ transition.
Case Receive**..** By the typability of P, we have that $P' = s?(x).Q \mid s[\mathtt{i} : v \cdot \vec{h}] \mid R \longrightarrow_s P'' = Q\{v/x\} \mid s[\mathtt{i} : \vec{h}] \mid R$.

From the induction step, we have that $P \approx P'$. To show that $P \approx P''$ we need to show that $P' \approx P''$. We will use the fact that bisimulation is a congruence. Consider $R \approx R$ and $s?(x).Q \mid s[\mathtt{i} : v \cdot \vec{h}] \approx Q \mid s[\mathtt{i} : \vec{h}]$.

Due to $s \notin \mathsf{fn}(R)$ we can compose in parallel bisimilar processes and get that $P' \approx P''$ as required.

The rest of the cases follow similar arguments. □

Proof for Lemma 3.9

Lemma C.5. *Let typable, localised P and actions ℓ_1, ℓ_2 such that $\mathsf{subj}(\ell_1), \mathsf{subj}(\ell_2)$ are session names and $\ell_1 \bowtie \ell_2$. If $\Gamma \vdash P \triangleright \Delta \stackrel{\ell_1}{\longrightarrow} P_1 \triangleright \Delta_1$ and $\Gamma \vdash P \triangleright \Delta \stackrel{\ell_2}{\longrightarrow} P_1 \triangleright \Delta_2$ then $\Gamma \vdash P_1 \triangleright \Delta \stackrel{\ell_2 \lfloor \ell_1}{\longrightarrow} P' \triangleright \Delta'$ and $\Gamma \vdash P_2 \triangleright \Delta \stackrel{\ell_1 \lfloor \ell_2}{\longrightarrow} P' \triangleright \Delta'$*

Proof. The result is an easy case analysis on all the possible combinations of ℓ_1, ℓ_2.

We give an interesting case. Let $(\nu\ a)(P \mid s_1[\mathtt{o}:\vec{h_1}\cdot a] \mid s_2[\mathtt{o}:\vec{h_2}\cdot a]) \xrightarrow{s_1!\langle a\rangle} P \mid s_1[\mathtt{o}:\vec{h_1}] \mid s_2[\mathtt{o}:\vec{h_2}\cdot a]$ and $(\nu\ a)(P \mid s_1[\mathtt{o}:\vec{h_1}\cdot a] \mid s_2[\mathtt{o}:\vec{h_2}\cdot a]) \xrightarrow{s_2!\langle a\rangle} P \mid s_1[\mathtt{o}:\vec{h_1}\cdot a] \mid s_2[\mathtt{o}:\vec{h_2}]$. Now it is easy to see that $P \mid s_1[\mathtt{o}:\vec{h_1}] \mid s_2[\mathtt{o}:\vec{h_2}\cdot a] \xrightarrow{s_2!\langle a\rangle} P \mid s_1[\mathtt{o}:\vec{h_1}] \mid s_2[\mathtt{o}:\vec{h_2}]$ and $P \mid s_1[\mathtt{o}:\vec{h_1}\cdot a] \mid s_2[\mathtt{o}:\vec{h_2}] \xrightarrow{s_1!\langle a\rangle} P \mid s_1[\mathtt{o}:\vec{h_1}] \mid s_2[\mathtt{o}:\vec{h_2}]$ as required. □

Two useful lemmas follow.

Lemma C.6. *Let P be session determinate. Then if $\Gamma \vdash P \triangleright \Delta \xrightarrow{\ell} P' \triangleright \Delta'$ and $\Gamma \vdash P \triangleright \Delta \xRightarrow{\ell} P'' \triangleright \Delta''$ then $P' \approx P''$*

Proof. There are two cases:
Case τ:. Follow Lemma 3.8 to get $P \approx P'$ and $P \approx P''$. The result then follows.

Case ℓ:. Suppose that $P \xrightarrow{\ell}_s P'$ and $P \xRightarrow{\ell}_s P''$ implies $P \Longrightarrow_s P_1 \xrightarrow{\ell}_s P_2 \Longrightarrow_s P''$. From Lemma 3.8, we can conclude that $P \approx P_1$ and because of the bisimulation definition, we have $P' \approx P_2$ to complete we call upon 3.8 once more to get $P' \approx P''$ as required. □

Lemma C.7. *Let P be session determinate and $\ell_1 \bowtie \ell_2$. Then if $\Gamma \vdash P \triangleright \Delta \xrightarrow{\ell_1} P_1 \triangleright \Delta_1$ and $\Gamma \vdash P \triangleright \Delta \xRightarrow{\ell_2} P_2 \triangleright \Delta_2$. Then $\Gamma \vdash P_1 \triangleright \Delta_1 \xRightarrow{\ell_2 \lfloor \ell_1} P' \triangleright \Delta'$ and $\Gamma \vdash P_2 \triangleright \Delta_2 \xRightarrow{\ell_1 \lfloor \ell_2} P'' \triangleright \Delta''$ and $P' \approx P''$*

Proof. The proof considers a case analysis on the combination of ℓ_1, ℓ_2.

Case $\ell_1 = s_1!\langle v_1\rangle, \ell_2 = s_2?\langle v_2\rangle$.

$$
\begin{aligned}
P \mid s_1[\mathtt{o}:\vec{h_1}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_2}] &\xrightarrow{\ell_1}_s P_1 \mid s_1[\mathtt{o}:\vec{h_1}] \mid s_2[\mathtt{i}:\vec{h_2}] \\
&\Longrightarrow_s P_1' \mid s_1[\mathtt{o}:\vec{h_1'}] \mid s_2[\mathtt{i}:\vec{h_2'}] \\
&\xrightarrow{\ell_2}_s P_1' \mid s_1[\mathtt{o}:\vec{h_1'}] \mid s_2[\mathtt{i}:\vec{h_2'}\cdot v_2] \\
&\Longrightarrow_s P' \mid s_1[\mathtt{o}:\vec{h_1''}] \mid s_2[\mathtt{i}:\vec{h_2''}] \\
P \mid s_1[\mathtt{o}:\vec{h_1}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_2}] &\Longrightarrow_s P_0 \mid s_1[\mathtt{o}:\vec{h_0}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_0'}] \\
&\xrightarrow{\ell_2}_s P_0' \mid s_1[\mathtt{o}:\vec{h_0}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_0'}\cdot v_2] \\
&\Longrightarrow_s P_2 \mid s_1[\mathtt{o}:\vec{h_2'}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_2''}\cdot v_2] \\
&\Longrightarrow_s P_2' \mid s_1[\mathtt{o}:\vec{h_3}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_3'}] \\
&\xrightarrow{\ell_2}_s P_2' \mid s_1[\mathtt{o}:\vec{h_4}] \mid s_2[\mathtt{i}:\vec{h_4'}] \\
&\Longrightarrow_s P'' \mid s_1[\mathtt{o}:\vec{h'}] \mid s_2[\mathtt{i}:\vec{h''}]
\end{aligned}
$$

By using Lemma 3.4, we have that $P \mid s_1[\mathtt{o}:\vec{h_1}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_2}] \Longrightarrow_s \xrightarrow{\ell_1}_s \xrightarrow{\ell_2}_s \Longrightarrow_s P' \mid s_1[\mathtt{o}:\vec{h_1''}] \mid s_2[\mathtt{i}:\vec{h_2''}]$ and $P \mid s_1[\mathtt{o}:\vec{h_1}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_2}] \xrightarrow{\ell_2}_s \Longrightarrow_s \xrightarrow{\ell_1}_s P'' \mid s_1[\mathtt{o}:\vec{h'}] \mid s_2[\mathtt{i}:\vec{h''}]$. We use the lemmas C.5, 3.4 to get $P \mid s_1[\mathtt{o}:\vec{h_1}\cdot v_1] \mid s_2[\mathtt{i}:\vec{h_2}] \xrightarrow{\ell_2}_s \Longrightarrow_s \xrightarrow{\ell_1}_s P' \mid s_1[\mathtt{o}:\vec{h_1''}] \mid s_2[\mathtt{i}:\vec{h_2''}]$.

The rest of the proof considers Lemma 3.8.

We summarise the above to prove Lemma 3.9.

Lemma. If P is session determinate then P is determinate and confluent.

Proof. From the definition of confluence (resp. determinacy) and from the definition of P we have that each derivative Q of P is also session determinate. The proof is an immediate result of lemmas C.7, resp. C.6.

Proof for Lemma 3.11

Lemma. Let $\mathscr{R}$ be a determinate upto-expansion relation. Then $\mathscr{R}$ is inside a bisimulation.

Proof. The proof is easy by showing $\Longrightarrow \mathscr{R} \Longleftarrow$ is a bisimulation. If we write this relation $\mathscr{S}$, we can easily check that this relation is a bisimulation, using determinacy (commutativity with other actions).

□

D Comparison with Asynchronous/Synchronous Calculi

D.1 Behavioural Theory for Session Type System with Input Buffer Endpoints

Before we prove the relations in § 5, we define a behavioural theory for the asynchronous session π-calculus with two end-point queues but without IO-queues [4, 5, 20].

$$\langle \mathsf{Acc}_A \rangle \quad a[\vec{s}] \xrightarrow{a\langle s\rangle} a[\vec{s}\cdot s] \quad \langle \mathsf{Req}_A \rangle \quad \overline{a}\langle s\rangle \xrightarrow{\overline{a}\langle s\rangle} \mathbf{0} \quad \langle \mathsf{In}_A \rangle \quad s[\vec{h}] \xrightarrow{s?\langle v\rangle} s[\vec{h}\cdot v]$$

$$\langle \mathsf{Out}_A \rangle \quad s!\langle v\rangle; P \xrightarrow{s!\langle v\rangle} P \quad \langle \mathsf{Bra}_A \rangle \quad s[\vec{h}] \xrightarrow{s\&l} s[\vec{h}\cdot l] \quad \langle \mathsf{Sel}_A \rangle \quad s!\lhd l; P \xrightarrow{s\oplus l} P$$

$$\langle \mathsf{Local}_A \rangle \frac{P \longrightarrow Q}{P \xrightarrow{\tau} Q} \quad \langle \mathsf{Par}_A \rangle \frac{P \xrightarrow{\ell} P' \ \mathsf{bn}(\ell)\cap \mathsf{fn}(Q)=\emptyset}{P \mid Q \xrightarrow{\ell} P' \mid Q} \quad \langle \mathsf{Tau}_A \rangle \frac{P \xrightarrow{\ell} P' \ Q \xrightarrow{\ell'} Q' \ \ell \asymp \ell'}{P \mid Q \xrightarrow{\tau} (\nu\ \mathsf{bn}(\ell,\ell'))(P' \mid Q')}$$

$$\langle \mathsf{Res}_A \rangle \frac{P \xrightarrow{\ell} P' \ n \notin \mathsf{fn}(\ell)}{(\nu\ n)P \xrightarrow{\ell} (\nu\ n)P'} \quad \langle \mathsf{OpenS}_A \rangle \frac{P \xrightarrow{\overline{a}\langle s\rangle} P'}{(\nu\ a)P \xrightarrow{\overline{a}(s)} P'}$$

$$\langle \mathsf{OpenN}_A \rangle \frac{P \xrightarrow{\overline{s}\langle a\rangle} P'}{(\nu\ a)P \xrightarrow{\overline{s}(a)} P'} \quad \langle \mathsf{Alpha}_A \rangle \frac{P \equiv_\alpha P' \ P' \xrightarrow{\ell} Q}{P \xrightarrow{\ell} Q}$$

Fig. 11. Labelled Transition for Session Type System with Two Buffer Endpoint Without IO

A labelled transition system is given in Figure 11. The LTS is similar to the LTS of the calculus studied in this paper (4), except from the output actions. Shared channels, and input actions have identical transition labels. The output actions cannot be observed on the buffer since there is no output buffer defined. Instead they are observed in an output reduction of a process.

D.2 Proofs for Section 5

We prove the results in § 5 for the two asynchronous session typed π-calculus, by either giving the bisimulation closures when a bisimulation holds or giving the counterexample when bisimulation does not hold. The results for the synchronous and asynchronous π-calculi are well-known, hence we omit.

1. **Case** $s!\langle v\rangle; s!\langle w\rangle; P \mid s\,[\mathtt{o}:\varepsilon] \not\approx s!\langle w\rangle; s!\langle v\rangle; P \mid s\,[\mathtt{o}:\varepsilon]$**.** On the left hand side process we can observe a τ transition and get $s!\langle w\rangle; P \mid s\,[\mathtt{o}:v] \xrightarrow{s!\langle v\rangle} s!\langle w\rangle; P \mid s\,[\mathtt{o}:\varepsilon]$ but $s!\langle w\rangle; s!\langle v\rangle; P \mid s\,[\mathtt{o}:\varepsilon] \stackrel{s!\langle v\rangle}{\not\Longrightarrow}$ as required.
2. **Case** $s_1!\langle v\rangle; s_2!\langle w\rangle; P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon] \approx s_2!\langle w\rangle; s_1!\langle v\rangle; P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon]$**.** Relation:

$$
\begin{aligned}
R = \{\ & (s_1!\langle v\rangle; s_2!\langle w\rangle; P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon]\ ,\ s_2!\langle w\rangle; s_1!\langle v\rangle; P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon]),\\
& (s_2!\langle w\rangle; P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:\varepsilon]\ ,\ P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:w]),\\
& (P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:w]\ ,\ s_1!\langle v\rangle; P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:w]),\\
& (P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:w]\ ,\ P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:w]),\\
& (s_2!\langle w\rangle; P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon]\ ,\ P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:w]),\\
& (P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon]\ ,\ P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:\varepsilon]),\\
& (P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:w]\ ,\ P \mid s_1\,[\mathtt{o}:\varepsilon] \mid s_2\,[\mathtt{o}:w]),\\
& (P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:\varepsilon]\ ,\ P \mid s_1\,[\mathtt{o}:v] \mid s_2\,[\mathtt{o}:\varepsilon])\}
\end{aligned}
$$

   gives the result.
3. **Case** $s?(x).s?(y).P \mid s\,[\mathtt{i}:\varepsilon] \not\approx s?(y).s?(x).P \mid s\,[\mathtt{i}:\varepsilon]$**.**
   On both processes we can observe a $s?\langle v\rangle$ transition and get $s?(x).s?(y).P \mid s\,[\mathtt{i}:v] \xrightarrow{\tau} s?(y).P\{v/x\} \mid s\,[\mathtt{i}:\varepsilon]$ and $s?(w).s?(v).P \mid s\,[\mathtt{i}:v] \xrightarrow{\tau} s?(x).P\{v/y\} \mid s\,[\mathtt{i}:\varepsilon]$. From the substitution, we have that both processes are not bisimilar.
4. **Case** $s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon] \approx s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]$**.**
   Relation

$$
\begin{aligned}
R = \{\ & (s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]\ ,\ s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:\varepsilon]\ ,\ s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:w]\ ,\ s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:w]),\\
& (s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:w]\ ,\ s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:w]),\\
& (s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]\ ,\ s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:w]\ ,\ s_1?(x).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:w]\ ,\ P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]\ ,\ s_1?(x).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (s_2?(y).P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:w]\ ,\ s_2?(y).s_1?(x).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:w]),\\
& (s_1?(x).s_2?(y).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:w]\ ,\ s_1?(x).P \mid s_1\,[\mathtt{i}:v] \mid s_2\,[\mathtt{i}:\varepsilon]),\\
& (P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon]\ ,\ P \mid s_1\,[\mathtt{i}:\varepsilon] \mid s_2\,[\mathtt{i}:\varepsilon])\}
\end{aligned}
$$

   gives the result.

D.3 Arrived Operators in the π-Calculi

In this subsection, we define two arrive inspected calculi that try to simulate blocking and order-preserving properties as the synchronous and asynchronous π-calculi, respectively.

For the synchronous π-calculus, we have blocking and order-preserving input and output and for the asynchronous π-calculus we have non-blocking and non-order preserving input and output.

In the context of the synchronous π-calculus, we cannot easily define the `arrived` operator without a slight compromise of the non blocking input property, due to the asynchronous nature of the `arrived` operator on the input queue of an endpoint.

The synchronous π-calculus can be represented asynchronously by having channel buffers of size one.

Syntax of the Synchronous-like π-Calculus with Arrive.

$$P ::= \mathbf{0} \mid a[\varepsilon] \mid a(x).P \mid \overline{a}\langle v\rangle.P \mid P|P \mid (\nu\, a)P \mid \texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, P$$

Labelled Transition Semantics of the Synchronous π-Calculus with Arrive.

$$\overline{a}\langle v\rangle.P \xrightarrow{\overline{a}\langle v\rangle} P \qquad a(x).P|a[\varepsilon] \xrightarrow{a\langle v\rangle} a(x).P|a[v] \qquad a(x).P \mid a[v] \longrightarrow P\{v/x\}$$

$$\frac{P \xrightarrow{\ell} P',\ \mathsf{fn}(\ell)\cap\mathsf{fn}(Q)=\emptyset}{P|Q \xrightarrow{\ell} P'|Q} \qquad \frac{P \xrightarrow{\ell} P',\ Q \xrightarrow{\ell'} Q',\ \ell \asymp \ell'}{P|Q \xrightarrow{\tau} (\nu\ \mathsf{bn}(\ell,\ell'))(P'|Q')} \qquad \frac{P \xrightarrow{\ell} P'\ n \notin \mathsf{fn}(\ell)}{(\nu\, n)P \xrightarrow{\ell} (\nu\, n)P'}$$

$$\frac{P \xrightarrow{\overline{a}\langle v\rangle} P'}{(\nu\, a)P \xrightarrow{\overline{a}\langle v\rangle} P'} \qquad \frac{P \equiv_\alpha P'\ P' \xrightarrow{\ell} Q}{P \xrightarrow{\ell} Q}$$

$$\texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, Q|a[\varepsilon] \xrightarrow{\tau} Q|a[\varepsilon]$$

$$\texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, Q|a[v] \xrightarrow{\tau} P|a[v]$$

In the synchronous π-calculus with `arrived` operator, channel buffers have size of one and we can receive a value from the environment, only if a corresponding process is ready to receive on the buffer channel.

To demonstrate the compromise done in to achieve this definition consider

$$a(x).P \quad \xrightarrow{a?\langle v\rangle} \quad P\{v/x\}$$

$$a(x).P \mid a[\varepsilon] \xrightarrow{a?\langle v\rangle} \longrightarrow P\{v/x\} \mid a[\varepsilon]$$

The first process is in the classic synchronous π calculus. We can only observe one input action. For the second system we observe an asynchronous input action. First a message is put in the communication buffer and then the actual receive happens. Between the two transition an arrive inspection can happen.

The asynchronous π-calculus with `arrived` operator is easier to be defined in a queue context. The idea here is to have endpoints that use a random policy for message exchange:

Syntax of the Asynchronous π-Calculus with Arrive.

$$P ::= \mathbf{0} \mid a[\varepsilon] \mid a(x).P \mid \overline{a}\langle v\rangle \mid P|P \mid (\nu\, a)P \mid \texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, P$$

Labelled Transition Semantics of the Asynchronous π-Calculus with Arrive.

$$\overline{a}\langle v\rangle \xrightarrow{\overline{a}\langle v\rangle} \mathbf{0} \qquad a[\vec{h}] \xrightarrow{a\langle h\rangle} a[\vec{h}\cdot h]$$

$$a?(x).P \mid a[\vec{h_1}\cdot h_i\cdot\vec{h_2}] \longrightarrow P\{h_i/x\} \mid a[\vec{h_1}\cdot\vec{h_2}] \qquad \frac{P \xrightarrow{\ell} P'\ \mathsf{fn}(\ell)\cap\mathsf{fn}(Q)=\emptyset}{P|Q \xrightarrow{\ell} P'|Q}$$

$$\frac{P \xrightarrow{\ell} P'\ Q \xrightarrow{\ell'} Q'\ \ell \asymp \ell'}{P|Q \xrightarrow{\tau} (\nu\ \mathsf{bn}(\ell,\ell'))(P'|Q')} \qquad \frac{P \xrightarrow{\ell} P'\ n \notin \mathsf{fn}(\ell)}{(\nu\, n)P \xrightarrow{\ell} (\nu\, n)P'}$$

$$\frac{P \xrightarrow{\overline{a}\langle v\rangle} P'}{(\nu\, v)P \xrightarrow{\overline{a}\langle v\rangle} P'} \qquad \frac{P \equiv_\alpha P'\ P' \xrightarrow{\ell} Q}{P \xrightarrow{\ell} Q}$$

$$\texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, Q|a[\varepsilon] \xrightarrow{\tau} Q|a[\varepsilon]$$

$$\texttt{if arrived}\, a\, \texttt{then}\, P\, \texttt{else}\, Q|a[h\cdot\vec{h}] \xrightarrow{\tau} P|a[\vec{h}]$$

The above definition disallows the order preserving property in the system but keeps the non blocking property as required by the asynchronous π-calculus.

Figure 6 shows that the arrive construct behaves the same on all of the calculi.

E Appendix for Selectors

E.1 Mapping

The selector operator is discussed in detailed along with its reduction properties and applications in [12]. In this section we present an ESP encoding of the selector.

We can extend ESP with the selector operations, with the following reduction semantics.

$$\begin{array}{l}
\texttt{new selector } r \texttt{ in } P \longrightarrow (\nu\, r)(P \mid \mathsf{sel}\langle r, \varepsilon\rangle) \qquad \texttt{register}\langle s', r\rangle; P \mid \mathsf{sel}\langle r, \vec{s}\rangle \longrightarrow P \mid \mathsf{sel}\langle r, \vec{s}\cdot s'\rangle \\
\texttt{let } x = \texttt{select}(r) \texttt{ in typecase } x \texttt{ of } \{(x_i : T_i) : P_i\}_{i\in I} \mid \mathsf{sel}\langle r, s'\cdot\vec{s}\rangle \mid s'[S, \texttt{i} : \vec{h}] \\
\qquad\qquad \longrightarrow P_i\{s'/x_i\} \mid \mathsf{sel}\langle r, \vec{s}\rangle \mid s'[S, \texttt{i} : \vec{h}] \quad (\vec{h} \neq \varepsilon) \\
\texttt{let } x = \texttt{select}(r) \texttt{ in typecase } x \texttt{ of } \{(x_i : T_i) : P_i\}_{i\in I} \mid \mathsf{sel}\langle r, s'\cdot\vec{s}\rangle \mid s'[\texttt{i} : \varepsilon] \\
\qquad\qquad \longrightarrow \texttt{let } x = \texttt{select}(r) \texttt{ in typecase } x \texttt{ of } \{(x_i : T_i) : P_i\}_{i\in I} \mid \mathsf{sel}\langle r, \vec{s}\cdot s'\rangle \mid s'[\texttt{i} : \varepsilon]
\end{array}$$

where in the third line S and T_i satisfies the condition for `typecase` in Figure 3. We also include the structural rules with garbage collection rules for queues as $(\nu\, r)\mathsf{sel}\langle r, \varepsilon\rangle \equiv \mathbf{0}$. Operator `new selector` r `in` P (binding r in P) creates a new selector $\mathsf{sel}\langle r, \varepsilon\rangle$, named r and with the empty queue ε. Operator $\texttt{register}\langle s', r\rangle; P$ registers a session channel s to r, adding s' to the original queue $\vec{s}$. $\texttt{let } x = \texttt{select}(r) \texttt{ in typecase } x \texttt{ of } \{(x_i : T_i) : P_i\}_{i\in I}$ retrieves a registered session and checks the availability to test if an event has been triggered. If so, find the match of the type of s' among $\{T_i\}$ and select P_i; if not, the next session is tested.

We now show this behaviour can be easily encoded by combining *arrival predicates* and *typecase*. Below we omit type annotations.

$$\begin{array}{l}
[\![\texttt{new selector } r \texttt{ in } P]\!] \stackrel{\text{def}}{=} (\nu\, b)(\overline{b}(r).b(\overline{r}).[\![P]\!] \mid b : [\varepsilon]) \quad [\![\texttt{register}\langle s, r\rangle; P]\!] \stackrel{\text{def}}{=} \overline{r}!\langle s\rangle; [\![P]\!] \\
[\![\texttt{let } x = \texttt{select}(r) \texttt{ in typecase } x \texttt{ of } \{(x_i : S_i) : P_i\}_{i\in I}]\!] \stackrel{\text{def}}{=} \mathsf{Select}(r\overline{r}) \\
[\![\mathsf{sel}\langle r, \vec{s}\cdot\vec{s}'\rangle]\!] \stackrel{\text{def}}{=} r[\texttt{o} : \vec{s}'] \mid \overline{r}[\texttt{i} : \vec{s}] \\
\texttt{def } \mathsf{Select}(x\overline{x}) = \overline{x}?(y); \texttt{if arrived } y \texttt{ then typecase } y \texttt{ of } \{(x_i : S_i) : [\![P_i]\!]\}_{i\in I} \\
\qquad\qquad \texttt{else } x!\langle y\rangle; \mathsf{Select}\langle x\overline{x}\rangle \qquad\qquad \texttt{in } \mathsf{Select}\langle r\overline{r}\rangle
\end{array}$$

The use of `arrived` is the key to avoid blocked inputs, allowing the system to proceed asynchronously. The operations on the collection need to carry session channels, hence the use of delegation (linear channel passing) is essential [8]. We can easily check that the embedding operationally simulates the selector given above as the extension of ESP, and that, under a suitable bisimulation, that it is semantically faithful.

E.2 Typing

The typing rules for the selector are naturally suggested from the ESP-typing of its encoding. We write the type for a *user* of a selector storing channels of type T, by $\overline{\texttt{sel}}(T)$, and the type for a selector itself by $\texttt{sel}(T)$. For simplicity we assume these types do not occur as part of other types. The linear environment Δ now includes two new type assignments, $r : \overline{\texttt{sel}}(T)$ and $r : \texttt{sel}(T)$. The typing rules for the selector follow.

$$\frac{\Gamma \vdash P \triangleright \Delta\cdot r : \overline{\texttt{sel}}(T)}{\Gamma \vdash \texttt{new selector } r \texttt{ in } TP \triangleright \Delta}(\mathsf{Selector}) \qquad \frac{\Gamma \vdash P \triangleright \Delta\cdot r : \overline{\texttt{sel}}(T) \quad S \leq T}{\Gamma \vdash \texttt{register}\langle s, r\rangle; P \triangleright \Delta\cdot r : \overline{\texttt{sel}}(T)\cdot s : S}(\mathsf{Register})$$

$$\frac{\forall i \in I.\Gamma \vdash P_i \triangleright \Delta\cdot r : \overline{\texttt{sel}}(T)\cdot x_i : S_i \quad S_i \leq T}{\Gamma \vdash \texttt{let } x = \texttt{select}(r) \texttt{ in typecase } x \texttt{ of } \{(x_i : S_i) : P_i\}_{i\in I} \triangleright \Delta\cdot r : \overline{\texttt{sel}}(T)}(\mathsf{Select})$$

The typing rule for the selector queue is similar to the runtime typing for a shared input queue. By setting $[\![\Delta]\!]$ as the compositional mapping such that $[\![r : \overline{\texttt{sel}}(T)]\!]$ is given as

$r : S_r \cdot \overline{r} : \overline{S_r}$ where $S_r = \mu \mathsf{X}.!(T);\mathsf{X}$, and otherwise identity, as well as extending the notion of error to the internal typecase of the select command, we obtain, writing ESP^+ for the extension of ESP with the selector:

Proposition E.1 (Soundness of Selector Typing Rules).

1. (Type Preservation) $\Gamma \vdash P \triangleright \Sigma$ *in* ESP^+ *if and only if* $\Gamma \vdash [\![P]\!] \triangleright [\![\Sigma]\!]$.
2. (Soundness) $P \equiv P'$ *implies* $[\![P]\!] \equiv [\![P']\!]$*; and* $P \longrightarrow P'$ *implies* $[\![P]\!] \longrightarrow^* [\![P']\!]$.
3. (Safety) *A typable process in* ESP^+ *never reduces to the error.*

Full proofs and further discussion are given in [12].

F Appendix: Lauer-Needham Transform

This appendix gives the detailed illustration of the Lauer-Needham transformation. Our translation uses the notations in Figure 12 for brevity, including: *pairs*, *polyadic inputs/outputs*, a *refined typecase*, a *refined selector*, and an *environment* as used in the standard CPS transform. All of them are easily encodable in the eventful calculus in [12].

$$
\begin{array}{rcl}
\mathscr{LN}[\![*a(w:S);P]\!] & \stackrel{\text{def}}{=} & (\nu o,q,\vec{c})(\ \mathsf{Loop}\langle o,q\rangle \mid \overline{o} \mid q\langle a,c_0\rangle \mid \mathsf{CodeBlocks}\langle a,o,q,\vec{c}\rangle\ ) \\
& & \quad \text{where } P_1,..,P_n \text{ are the positive sub-terms of } P;\ P_1,..,P_{n-m} \text{ the} \\
& & \quad \text{blocking ones whose subjets are respectively typed } S_1,..,S_{n-m}; \\
& & \quad \text{and } o,\ q \text{ and } \vec{c} = c_1..c_n \text{ are fresh and pairwise distinct.} \\
\mathsf{Loop}\langle o,q\rangle & \stackrel{\text{def}}{=} & *o.\ \mathtt{let}\ w = \mathtt{select}(q)\ \mathtt{in}\ \mathtt{typecase}\ w\ \mathtt{of}\ \{ \\
& & \qquad (x:S,\ z:\mathsf{env}):\ \mathtt{new}\ y:\mathtt{env}\ \mathtt{in}\ \overline{z}\langle y\rangle, \\
& & \qquad (x:S_1,\ y:\mathsf{env},\ z:S_1,\mathsf{env}):\ \overline{z}\langle x,y\rangle,\ \ldots \\
& & \qquad (x:S_{n-m},y:\mathsf{env},z:S_{n-m},\mathsf{env}):\ \overline{z}\langle x,y\rangle \\
& & \quad \} \\
\mathsf{CodeBlocks}\langle a,o,q,\vec{c}\rangle & \stackrel{\text{def}}{=} & \mathscr{B}[\![a(w:S);P]\!] \mid \prod_{1\le i\le n}\mathscr{B}[\![P_i]\!] \\
\mathscr{B}[\![*a(w:S).P]\!] & \stackrel{\text{def}}{=} & *c_0(y).a(w':S).\,\mathtt{update}(y,w,w');\,\mathtt{register}\langle q,a,c_0\rangle;[\![P,y]\!] \\
\mathscr{B}[\![x^{(i)}?(z:T).Q]\!] & \stackrel{\text{def}}{=} & *c_i(x',y).x'?(z').\,\mathtt{update}(y,z,z');\,\mathtt{update}(y,x,x');\,[\![Q,y]\!] \\
\mathscr{B}[\![x^{(i)} \triangleright \{l_j:Q_j\}_j]\!] & \stackrel{\text{def}}{=} & *c_i(x',y).x' \triangleright \{l_j:\ \mathtt{update}(y,x,x');\,[\![Q_j,y]\!]\}_j \\
[\![x!\langle e\rangle;Q,y]\!] & \stackrel{\text{def}}{=} & \mathtt{let}\ x' = [\![x]\!]_y\ \mathtt{in}\ x'!\langle [\![e]\!]_y\rangle;\mathtt{update}(y,x,x');\,[\![Q,y]\!] \\
[\![x!\langle k\rangle;Q,y]\!] & \stackrel{\text{def}}{=} & \mathtt{let}\ x' = [\![x]\!]_y\ \mathtt{in}\ \mathtt{let}\ k' = [\![k]\!]_y\ \mathtt{in}\ x'!\langle k'\rangle;\mathtt{update}(y,xk,x'k');\,[\![Q,y]\!] \\
[\![x \triangleleft l_j;Q,y]\!] & \stackrel{\text{def}}{=} & \mathtt{let}\ z = [\![x]\!]_y\ \mathtt{in}\ z \triangleleft l_j;[\![Q,y]\!] \\
[\![\overline{b}(z:S);Q,y]\!] & \stackrel{\text{def}}{=} & \overline{b}(z':S);\mathtt{update}(y,z,z');\,[\![Q,y]\!] \\
[\![Q,y]\!] & \stackrel{\text{def}}{=} & \mathtt{let}\ x' = [\![x]\!]_y\ \mathtt{in}\ \mathtt{register}\langle q,x',c_i,y\rangle;\overline{o}\ \ (Q \text{ is blocking at } x^{(i)}) \\
[\![\mathbf{0},y]\!] & \stackrel{\text{def}}{=} & \overline{o}
\end{array}
$$

Fig. 12. Translation Function for Lauer-Needham Transform

The formal mapping follows. Below we say a process is *positive* if it is either an acceptor, an input, a branching or a definition, and is *blocking* if it is positive and is not a definition. The *subject* of a positive term is the initial channel name if it is blocking, and the initial process variable if it is a definition.

Definition F.1 (Lauer-Needham Transform). Let $*a(w:S);P$ be a simple server. Then the mapping $\mathscr{LN}[\![*a(w:S);P]\!]$ is inductively defined by the rules in Figure 12, assuming the following annotation on P: the subjects of distinct positive subterms in P are labelled with distinct numerals from 1 to n, as in e.g. $x^{(i)}?(y);Q$ (then we say the prefix is *blocking at* $x^{(i)}$), such that 1 to $n-m$ are used for the prefixes and the rest for the definitions ($n \geq m \geq 0$). We also assume all environments have the same fields named by the (free and bound) variables occurring in P which we assume to be pairwise distinct.

As outlined in the main section, the main map $\mathscr{LN}[\![*a(w:S);P]\!]$ consists of:

1. An *event loop* $\mathsf{Loop}\langle o,q\rangle$ which denotes a loop invoked at o without parameters. It also uses a selector queue q. It is composed with $\overline{o}$, initiating the loop.
2. A selector queue $q\langle a,c_0\rangle$ named q with a single element $\langle a,c_0\rangle$.
3. A collection of *code blocks* $\mathsf{CodeBlocks}\langle a,o,q,\vec{c}\rangle$, each defined using an auxiliary map $\mathscr{B}[\![R]\!]$ and $[\![Q,y]\!]$. Its behaviour is illustrated below.

The initial execution of $\mathscr{LN}[\![*a(w:S);P]\!]$ starts from the *event-loop*. It fetches a channel at which a message has arrived by *select*: what it finds in the selector queue is checked and typed by the *typecase* construct from [12] (as illustrated in Appendix A). Initially it will only find a request via a. After finding it, the loop then creates a brand new environment and jumps to the *initial code block* at c_0, passing the environment.

Once invoked, the initial code block, $\mathscr{B}[\![a(w:S);P]\!]$, receives a fresh session channel through the buffer of a, saves it in the environment, and moves to $[\![P,y]\!]$. The code $[\![P,y]\!]$ carries out "instructions" from P, using the environment denoted by y to interpret variables. After completing all the consecutive *non-blocking actions* (invocations, outputs, selections, conditionals and recursions) starting from the initial input, the code will reach a blocking prefix or $\mathbf{0}$. If the former is the case, it registers that blocking session channel, the associated continuation and the current environment in a *selector queue*. Then the control flow returns to the *event loop*.

The event loop then tries to sense the arrival of a message again by scanning the registered channels (shared and session). Assume it finds a message via a session channel this time. It then decides its type by *typecase* and invokes the corresponding continuation code block, passing the session channel and the environment. The code block, which has the shape $\mathscr{B}[\![P_i]\!]$ for a blocking sub-term P_i of P, now receives the message via the passed session channel, saves it in the passed environment, and continues with the remaining behaviour until it reaches a blocking action, in the same way as illustrated for the initial code block. The combination of a typecase and a session channel passing above enables the protection of session type abstraction, ensuring type and communication safety.

G LN Transform Properties

In this section consider that $B_1 = s_1[\mathtt{i}:\vec{h}_1,\mathtt{o}:\vec{h}'_1]$

G.1 Selector Properties

Lemma G.1. *Let*

$$
\begin{array}{l}
Q_i = \mathbf{Def} \\
\qquad\qquad \mathbf{X_1} = \mathtt{if\ arrive}\ s_1\ \mathtt{then}\ C_1[\mathbf{X_2}]\ \mathtt{else}\ \mathbf{X_2} \\
\qquad\qquad\quad \vdots \\
\qquad\qquad \mathbf{X_n} = \mathtt{if\ arrive}\ s_n\ \mathtt{then}\ C_n[\mathbf{X_1}]\ \mathtt{else}\ \mathbf{X_1} \\
\mathbf{in}\ \mathbf{X_i}
\end{array}
$$

with $C_i = \texttt{typecase}\ s_i\ \textit{of}\ \{S_1 : R_{i1}; -, \ldots, S_m : R_{im}; -\}$

$$P = \texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \texttt{typecase}\ x\ \textit{of}\ \{S_1 : C[R_1], \ldots . S_m : C[R_m]\} \mid r\langle s_1, \ldots, s_n\rangle$$

Recall that the selector recurses on a process variable (appendix E). In this case the process variable is $\mathbf{X}$*. Now we define C.*
$C = -; \texttt{register}\ x\ \texttt{to}\ r\ \texttt{in}\ \mathbf{X}$ *and* $R_{ij} = R_j\{s_i/x\}$.
Then $Q_1 \mid s_1[\texttt{i} : \vec{h_1}, \texttt{o} : \vec{h'_1}] \mid \ldots \mid s_n[\texttt{i} : \vec{h_n}, \texttt{o} : \vec{h'_n}] \approx P \mid s_1[\texttt{i} : \vec{h_1}, \texttt{o} : \vec{h'_1}] \mid \ldots \mid s_n[\texttt{i} : \vec{h_n}, \texttt{o} : \vec{h'_n}]$

Proof. By unfolding P n times we have a process that can be related to $Q_1 \mid B_1 \mid \ldots \mid B_n$. To relate the two processes we use the selector definition for select and register. From there it easy to build and verify a bisimulation closure.

Lemma G.2. $\texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \texttt{typecase}\ x\ \textit{of}\ \{S_1 : C[R_1], \ldots . S_m : C[R_m]\} \mid r\langle s_1, \ldots, s_n\rangle \mid s_1[\texttt{i} : h_1, \texttt{o} : h'_1] \mid \ldots \mid s_n[\texttt{i} : h_n, \texttt{o} : h'_n]$ *is confluent.*
Where $C = -; \texttt{register}\ x\ \texttt{to}\ r\ \texttt{in}\ \mathbf{X}$, $C[R_i]$ *is sequential and* $C[R_i]\{s_j/x\} \mid s_j[\texttt{i} : \vec{h_j}, \texttt{o} : \vec{h'_j}]$ *is session determinate and non blocking.*

Proof. The fact that the selector is reduced using the rule [Sarr] makes the selector process to be non session determinate. We will need to use the confluence definition to establish the final result.
We build our result out of simpler results.
Let

$$\begin{aligned} Q_i = \textbf{Def}\ & \\ & \mathbf{X_1} = \texttt{if arrive}\ s_1\ \texttt{then}\ C_1[\mathbf{X_2}]\ \texttt{else}\ \mathbf{X_2} \\ & \mathbf{X_2} = \texttt{if arrive}\ s_2\ \texttt{then}\ C_2[\mathbf{X_1}]\ \texttt{else}\ \mathbf{X_1} \\ \textbf{in}\ \mathbf{X_i} & \end{aligned}$$

and define $s[\texttt{i} : \vec{h'_i} \cdot \vec{h_i}, \texttt{o} : \vec{h_o}] \succ s[\texttt{i} : \vec{h_i}, \texttt{o} : \vec{h'_o} \cdot \vec{h_o}]$
We show that $Q_1 \mid B_1 \mid B_2$ is confluent. We will use confluence definition to get a derivative of $Q_1 \mid B_1 \mid B_2$.

$$Q_1 \mid B_1 \mid B_2 \stackrel{\vec{l}}{\Longrightarrow} Q = R \mid B'_1 \mid B'_2$$

We do a case analysis on all possible combinations of ℓ_1, ℓ_2. We will show a case.
$Q \stackrel{s_1?\langle v\rangle s_2!\langle v\rangle}{\longrightarrow\!\!\!\!\Longrightarrow} R \mid s_1[\texttt{i} : \vec{h_1} \cdot v] \mid s_2[\texttt{i} : \vec{h_2}, \texttt{o} : \vec{h'_2}]$, $Q \stackrel{s_2!\langle v\rangle s_1?\langle v\rangle}{\Longrightarrow\!\!\!\!\longrightarrow} R' \mid s_1[\texttt{i} : \vec{h''_1}] \mid s_2[\texttt{i} : \vec{h''_2}, \texttt{o} : \vec{h'''_2}]$.
We show that the above resulting processes are bisimilar.

$$\begin{aligned} \mathscr{R} = \{ & (P,Q), (Q,P) \mid \\ & P = R'' \mid B_1 \mid B_2 \\ & Q = Q_1 \mid B'_1 \mid B'_2 \\ & \text{where } R'' \text{ is a postfix of processes } C_i[X_j] \\ & B_i \succ B'_i\} \end{aligned}$$

Lemma G.1 shows that $\texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \texttt{typecase}\ x\ \text{of}\ \{S_1 : C[R'_1], S_2 : C[R'_2]\} \mid r\langle s_1, s_2\rangle \mid s_1[\texttt{i} : h_1, \texttt{o} : h'_1] \mid s_2[\texttt{i} : h_2, \texttt{o} : h'_2]$ is also confluent since bisimulation preserves confluency.
We can now generalise for the n-ary case. The bisimulation closure is similar to the binary case.

$$\begin{aligned}\mathscr{R} = \{&(P,Q),(Q,P) \mid \\ &P = R'' \mid B_1 \mid \ldots \mid B_n \\ &Q = Q_1 \mid B_1' \mid \ldots \mid B_n' \\ &\text{where } R'' \text{ is a postfix of processes } C_i[X_j] \\ &B_i \succ B_i'\}\end{aligned}$$

The rest of the proof is similar to the binary case.

We give a core result about the selector's behaviour. The selector's ordering of events is irrelevant to the selector's behaviour when each events computation follows a confluent execution.

Lemma G.3. $\texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \textit{typecase}\ x\ \textit{of}\ \{S_1 : C[R_1], \ldots S_m : C[R_m]\} \mid r\langle s_1, \ldots, s_i, s_{i+1}, \ldots, s_n\rangle \mid s_1[\texttt{i} : h_1, \texttt{o} : h_1'] \mid \ldots \mid s_n[\texttt{i} : h_n, \texttt{o} : h_n'] \approx \texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \textit{typecase}\ x\ \textit{of}\ \{S_1 : C[R_1], \ldots S_m : C[R_m]\} \mid r\langle s_1, \ldots, s_{i+1}, s_i, \ldots, s_n\rangle \mid s_1[\texttt{i} : h_1, \texttt{o} : h_1'] \mid \ldots \mid s_n[\texttt{i} : h_n, \texttt{o} : h_n']$
Where $C = -; \texttt{register}\ x\ \texttt{to}\ r\ \texttt{in}\ \mathbf{X}$, $C[R_i]$ *is sequential and* $C[R_i]\{s_j/x\} \mid s_j[\texttt{i} : \vec{h_j}, \texttt{o} : \vec{h_j'}]$ *is session determinate and non blocking.*

Proof. We will construct a bisimulation relying on simpler bisimilar processes. Let

$$\begin{aligned}&Q_i = \textbf{Def} \\ &\qquad \mathbf{X_1} = \texttt{if arrive}\ s_1\ \texttt{then}\ s_1?(x); \mathbf{X_2}\ \texttt{else}\ \mathbf{X_2} \\ &\qquad \mathbf{X_2} = \texttt{if arrive}\ s_2\ \texttt{then}\ s_2?(x); \mathbf{X_1}\ \texttt{else}\ \mathbf{X_1} \\ &\textbf{in}\ X_i\end{aligned}$$

and

$$B_i = s_i[\texttt{i} : \vec{h_i}, \texttt{o} : \vec{h_i'}]$$

We give a confluent up-to relation.
Let $\mathscr{S} = \{(Q_1 \mid B_1 \mid B_2, Q_2 \mid B_1 \mid B_2), (Q_1 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon], Q_1 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon]), (Q_2 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon], Q_2 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon])\}$.
Simple transitions give the required result. Both processes are confluent from proof of lemma G.1.
Now let

$$\begin{aligned}&Q_i = \textbf{Def} \\ &\qquad \mathbf{X_1} = \texttt{if arrive}\ s_1\ \texttt{then}\ C_1[\mathbf{X_2}]\ \texttt{else}\ \mathbf{X_2} \\ &\qquad \mathbf{X_2} = \texttt{if arrive}\ s_2\ \texttt{then}\ C_2[\mathbf{X_1}]\ \texttt{else}\ \mathbf{X_1} \\ &\textbf{in}\ \mathbf{X_i}\end{aligned}$$

with $C_i = \texttt{typecase}\ s_i\ \text{of}\ \{S_1 : R_{i1}; -, \ldots, S_m : R_{im}; -\}$.
Let

$$\begin{aligned}\mathscr{S} = \{&(Q_1 \mid B_1 \mid B_2, Q_2 \mid B_1 \mid B_2), \\ &(Q_1 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon], Q_1 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon]), \\ &(Q_2 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon], Q_2 \mid s_1[\texttt{i} : \varepsilon] \mid s_2[\texttt{i} : \varepsilon])\}\end{aligned}$$

Again we verify with simple transitions that $\mathscr{S}$ is closed under the bisimulation definition.
Now let

$$R = \texttt{typecase}\ x\ \text{of}\ \{S_1 : C[R_1], S_2 : C[R_2]\}$$

with $C = -; \texttt{register}\ x\ \texttt{to}\ r\ \texttt{in}\ \mathbf{X}$ and $R_{ij} = R_j\{s_i/x\}$. and

$P_1 = \mu X.r?(y);\texttt{if arrive } y \texttt{ then } R \texttt{ else } \overline{r}!\langle x\rangle;\mathbf{X} \mid r[\texttt{i} : s_1, s_2, \texttt{o} : \varepsilon] \mid \overline{r}[\texttt{i} : \varepsilon, \texttt{o} : \varepsilon] \mid B_1 \mid B_2$
$Q_1 = \mu X.r?(y);\texttt{if arrive } y \texttt{ then } R \texttt{ else } \overline{r}!\langle x\rangle;\mathbf{X} \mid r[\texttt{i} : s_2, s_1, \texttt{o} : \varepsilon] \mid \overline{r}[\texttt{i} : \varepsilon, \texttt{o} : \varepsilon] \mid B_1 \mid B_2$

We show that $P_1 \approx Q_1$ by using lemma G.1 to relate under bisimulation $\mathbf{X_1} \mid B_1 \mid B_2 \approx PP_1$ and $\mathbf{X_2} \mid B_1 \mid B_2 \approx Q_1$.
We generalize for the n-ary case.
Define

$$
\begin{aligned}
QY_i = \textbf{Def}\ & \\
& \mathbf{Y_1} = \texttt{if arrive } s_1 \texttt{ then } C_1[\mathbf{Y_2}] \texttt{ else } \mathbf{Y_2} \\
& \vdots \\
& \mathbf{Y_i} = \texttt{if arrive } s_i \texttt{ then } C_i[\mathbf{Y_{i+1}}] \texttt{ else } \mathbf{Y_{i+1}} \\
& \mathbf{Y_{i+i}} = \texttt{if arrive } s_{i+1} \texttt{ then } C_{i+1}[\mathbf{Y_{i+2}}] \texttt{ else } \mathbf{Y_{i+2}} \\
& \vdots \\
& \mathbf{Y_n} = \texttt{if arrive } s_n \texttt{ then } C_n[\mathbf{Y_1}] \texttt{ else } \mathbf{Y_1} \\
\textbf{in } \mathbf{Y_i} &
\end{aligned}
$$

$$
\begin{aligned}
QX_i = \textbf{Def}\ & \\
& \mathbf{X_1} = \texttt{if arrive } s_1 \texttt{ then } C_1[\mathbf{X_2}] \texttt{ else } \mathbf{X_2} \\
& \vdots \\
& \mathbf{X_{i-i}} = \texttt{if arrive } s_{i-i} \texttt{ then } C_{i-i}[\mathbf{X_{i+1}}] \texttt{ else } \mathbf{X_{i+1}} \\
& \mathbf{X_{i+i}} = \texttt{if arrive } s_{i+i} \texttt{ then } C_{i+i}[\mathbf{X_i}] \texttt{ else } \mathbf{X_i} \\
& \mathbf{X_i} = \texttt{if arrive } s_i \texttt{ then } C_i[\mathbf{X_{i+2}}] \texttt{ else } \mathbf{X_{i+2}} \\
& \vdots \\
& \mathbf{X_n} = \texttt{if arrive } s_n \texttt{ then } C_n[\mathbf{X_1}] \texttt{ else } \mathbf{X_1} \\
\textbf{in } \mathbf{X_i} &
\end{aligned}
$$

We again construct a simple confluent up-to bisimulation, given the fact that $PX_1 \mid B_1 \mid \ldots \mid B_n, PY_1 \mid B_1 \mid \ldots \mid B_n$ are confluent processes.

$$\mathscr{R} = \{(PX_1 \mid B_1 \mid \ldots \mid B_n, PY_1 \mid B_1 \mid \ldots \mid B_n)\}$$

We verify this relation using simple transitions. Processes $C_i[\mathbf{Y_{i+1}}], C_i[\mathbf{X_{i+1}}]$ are non blocking and can be reduced by observing τ transitions.
Now lets take the processes:

$$
\begin{aligned}
P &= \texttt{select } x \texttt{ from } r \texttt{ in } R \mid r\langle s_1, \ldots, s_i, s_{i+1}, \ldots, s_n\rangle \mid B_1 \mid \ldots \mid B_n \\
Q &= \texttt{select } x \texttt{ from } r \texttt{ in } R \mid r\langle s_1, \ldots, s_{i+1}, s_i, \ldots, s_n\rangle \mid B_1 \mid \ldots \mid B_n
\end{aligned}
$$

Again lemma G.1 is used to relate under bisimulation $P \approx PY_1 \mid B_1 \mid \ldots \mid B_n$ and $Q \approx PX_1 \mid B_1 \mid \ldots \mid B_n$ as we did in the binary case.

The above lemma gives an understanding about the selectors behaviour when dealing with a static set of arrived-inspect sessions. When we add the arrive-inspection of a shared queue, we can dynamically add session queues in the selector (as in the LN-transform). This fact should not change the selector's behaviour.

Lemma G.4. $\texttt{select } x \texttt{ from } r \texttt{ in } \textit{typecase } x \textit{ of } \{S_1 : C[R_1], \ldots . S_m : C[R_m]\} \mid r\langle a, s_1, \ldots, s_i, s_{i+1}, \ldots, s_n\rangle \mid s_1[\texttt{i} : h_1, \texttt{o} : h_1'] \mid \ldots \mid s_n[\texttt{i} : h_n, \texttt{o} : h_n'] \mid a[\vec{s}]$

$\approx$ `select` x `from` r `in` *typecase* x *of* $\{S_1 : C[R_1], \ldots . S_m : C[R_m]\} \mid r\langle a, s_1, \ldots, s_{i+1}, s_i, \ldots, s_n\rangle \mid s_1[\mathtt{i} : h_1, \mathtt{o} : h_1'] \mid \ldots \mid s_n[\mathtt{i} : h_n, \mathtt{o} : h_n'] \mid a[\vec{s}]$
Where $C[R_i] = -$; `register` x `to` r `in` $\mathbf{X}$, $C[R_i]$ *is sequential and* $C[R_i]\left\{s_j/x\right\} \mid s_j[\mathtt{i} : \vec{h_j}, \mathtt{o} : \vec{h_j'}]$ *is session determinate and non blocking.*

Proof. The proof continues the proof from G.3.
We create an up-to confluent relation for the dynamic selector.

$$\begin{aligned}\mathscr{R} = \{&(\texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \texttt{typecase}\ \ x\ \text{of}\ \{S_1 : C[R_1], \ldots . S_m : C[R_m]\} \mid \\ &r\langle a, s_1, \ldots, s_i, s_{i+1}, \ldots\rangle \mid s_1[\mathtt{i} : h_1, \mathtt{o} : h_1'] \mid \ldots \mid a[\vec{s}], \\ &\texttt{select}\ x\ \texttt{from}\ r\ \texttt{in}\ \texttt{typecase}\ \ x\ \text{of}\ \{S_1 : C[R_1], \ldots . S_m : C[R_m]\} \mid \\ &r\langle a, s_1, \ldots, s_{i+i}, s, \ldots\rangle \mid s_1[\mathtt{i} : h_1, \mathtt{o} : h_1'] \mid \ldots \mid a[\vec{s}])\}\end{aligned}$$

Note that the selector can have an arbitrary numbers of sessions registered to it, along with the corresponding session endpoint. The verification uses simple reduction. The interesting case is when a new session is created and registered in the selector.

G.2 LN-transform properties

Lemma G.5. $*(a(x).P) \mid a[\varepsilon]$, *where P is session determinate and sequential, is confluent.*

Proof. Since $a(x).P \mid a[s \cdot \vec{s}] \longrightarrow P\{s/x\} \mid s[\mathtt{i} : \varepsilon, \mathtt{o} : \varepsilon] \mid a[s \cdot \vec{s}]$ and $P\{s/x\}$ is confluent by theorem **??**. So it is trivial to see that $a(x).P \mid a[s \cdot \vec{s}]$ is also confluent.
We want to show that $Q = (\mu\mathbf{X}.a(x).P \mid \mathbf{X}) \mid a[\vec{s}]$ is confluent.
For a derivative of Q we have $(\mu\mathbf{X}.a(x).P \mid \mathbf{X}) \mid a[\vec{s}] \mid R_1 \mid \ldots \mid R_n \mid B_1 \mid \ldots B_n$ and $R_i \mid B_i$ are session determinate.
We should verify the confluence definition. We can do a case analysis on all possible action definitions to get that the entire process is confluence.

To work with the event based server we establish an equivalence result between CPS style and recursive progamming style.

Lemma G.6.

$$\begin{aligned}&\textbf{Def}\\ &\quad \mathbf{X_1} = C_1[X_i]\\ &\quad \mathbf{X_2} = C_2[X_j]\\ &\quad \vdots\\ &\quad \mathbf{X_n} = C_n[X_k]\\ &\mathtt{in} \qquad \mathbf{X_m} \mid \textstyle\prod_{l\in J} s_l[\mathtt{i} : \vec{h_l}, \mathtt{o} : \vec{h_l'}]\end{aligned}$$

$\approx Q = *c_1.C_1[![c_i]] \mid *c_2.C_2[![c_j]] \mid \ldots \mid *c_n.C_1[![c_k]] \mid ![c_m] \mid \prod_{l\in J} s_l[\mathtt{i} : \vec{h_l}, \mathtt{o} : \vec{h_l'}]$
where $C_i = R_i; -$ *and* R_i *is sequential (does not contain parallel composition, restriction or if-else construct) and non-blocking (input prefixes can immediately receive a value from a queue)*

Proof. We can create a closed relation under bisimulation.

$$\begin{aligned}\mathscr{R} = \{P_1, P_2 \mid\ & P_1 = \textbf{Def}\ \ \mathbf{X_1} = C_1[X_i], \ldots, \mathtt{in}\ \mathbf{X}_i\\ & P_2 = *c_1.C_1[![c_i]] \mid \ldots \mid C_i[![c_j]]\\ & P_1 = \textbf{Def}\ \ \mathbf{X_1} = C_1[X_i], \ldots, \mathtt{in}\ R_i'\\ & P_2 = *c_1.C_1[![c_i]] \mid \ldots \mid R_i''\\ & C_i[X_j] \Longrightarrow R_i', C_i[![c_j]] \Longrightarrow R_i''\}\end{aligned}$$

The fact that $\mathscr{R}$ is a bisimulation can be verified using simple transitions.

A usefull definition is that of the LN-transform in recursive programming style.

Definition G.1 (LN transform-recursive programming style).

$$
\begin{aligned}
LNR[\![*a(x).P]\!] &\stackrel{def}{=} (\nu\ q)(\mathsf{Loop}\langle q\rangle \mid q\langle a\rangle) \\
&\quad \textit{where } P_1,..,P_n \textit{ are the positive sub-terms of } P;\ P_1,..,P_{n-m} \textit{ the} \\
&\quad \textit{blocking ones whose subjets are respectively typed } S_1,..,S_{n-m}; \\
&\quad \textit{and } o,\ q \textit{ and } \vec{c} = c_1..c_n \textit{ are fresh and pairwise distinct.} \\
\mathsf{Loop}\langle q\rangle &\stackrel{def}{=} \mathtt{select}\ x\ \mathtt{from}\ q\ \mathtt{in}\ \textit{typecase}\ x\ \textit{of}\ \{ \\
&\qquad (x:S) : \mathtt{new}\ y:\mathtt{env}\ \mathtt{in}\ )\mathscr{B}[\![*a(x).P]\!], \\
&\qquad (x:S_1,\ y:\mathsf{env}) :\ \mathscr{B}[\![P_1]\!],\ \ldots \\
&\qquad (x:S_{n-m},\ y:\mathsf{env}) :\ \mathscr{B}[\![P_n]\!] \\
&\quad \} \\
\mathscr{B}[\![*a(x).P]\!] &\stackrel{def}{=} a(w).\mathtt{update}(y,w,w');\ \mathtt{register}\ a\ \mathtt{to}\ q\ \mathtt{in}\ [\![P,y]\!] \\
\mathscr{B}[\![x^{(i)}?(z:T);Q]\!] &\stackrel{def}{=} x'?(z');\ \mathtt{update}(y,z,z');\ \mathtt{update}(y,x,x');\ [\![Q,y]\!] \\
\mathscr{B}[\![x^{(i)}\&\{l_j:Q_j\}_j]\!] &\stackrel{def}{=} x'\&\{l_j:\ \mathtt{update}(y,x,x');\ [\![Q_j,y]\!]\}_j \\
[\![x!\langle e\rangle;Q,y]\!] &\stackrel{def}{=} \mathtt{Let}\ x' = [\![x]\!]_y\ \mathtt{in}\ \mathtt{in}\ x'!\langle [\![e]\!]_y\ \mathtt{in}\ \rangle;\ \mathtt{update}(y,x,x');\ [\![Q,y]\!] \\
[\![x!\langle k\rangle;Q,y]\!] &\stackrel{def}{=} \mathtt{Let}\ x' = [\![x]\!]_y\ \mathtt{in}\ \mathtt{in}\ \mathtt{Let}\ k' = [\![k]\!]_y\ \mathtt{in}\ \mathtt{in}\ x'!\langle k'\rangle;\ \mathtt{update}(y,xk,x'k');\ [\![Q,y]\!] \\
[\![x\oplus l_j;Q,y]\!] &\stackrel{def}{=} \mathtt{Let}\ z = [\![x]\!]_y\ \mathtt{in}\ \mathtt{in}\ z\oplus l_j;\ [\![Q,y]\!] \\
[\![\overline{b}(z:S);Q,y]\!] &\stackrel{def}{=} \overline{b}(z':S);\mathtt{update}(y,z,z');\ [\![Q,y]\!] \\
[\![Q,y]\!] &\stackrel{def}{=} \mathtt{Let}\ x' = [\![x]\!]_y\ \mathtt{in}\ \mathtt{in}\ \mathtt{register}\ x',y\ \mathtt{to}\ q\ \mathtt{in}\ \mathsf{Loop}\langle q\rangle\ \ (Q \textit{ is blocking at } x^{(i)}) \\
[\![\mathbf{0},y]\!] &\stackrel{def}{=} \mathsf{Loop}\langle q\rangle
\end{aligned}
$$

Lemma G.7. $LN[\![*(a(x).P) \mid a[\varepsilon]]\!] \approx LNR[\![*(a(x).P) \mid a[\varepsilon]]\!]$

Proof. The proof is a direct application of lemma G.6.

The LN-transformed process is essentially a sequential process with session endpoints composed in parallel. Hence we can also establish:

Lemma G.8. $LN[\![*(a(x).P) \mid a[\varepsilon]]\!]$ *is confluent.*

Proof. We use lemma G.2 to show that $LNR[\![*(a(x).P) \mid a[\varepsilon]]\!]$ then by lemma G.7 and the fact that bisimulation preserves confluence, we get the required result.

Using these results, we can establish the fundamental result, which crucially relies on asynchronous nature of our bisimulations, noting that the original simple server and its translation generally have completely different timings at which the session input/branching subjects (prefixes) become active: in the former they are made active in parallel for all threads, while in the latter they are made active only sequentially.

We can know study the behaviour of the LN-transform.

Lemma G.9. *Let*

$$P_1 = (\nu\ \vec{c}or)(\mathsf{Loop} \mid \mathsf{CodeBlocks} \mid r\langle\ldots,(s_i,\xi_i,c_i),(s_j,\xi_j,c_j),\ldots\rangle \mid a[\vec{s}] \mid \prod_{m\in I} s_m[\mathtt{i}:\vec{h_{im}},\mathtt{o}:\vec{h_{om}}]),$$

$$P_2 = (\nu\ \vec{c}or)(\mathsf{Loop} \mid \mathsf{CodeBlocks} \mid r\langle\ldots,(s_j,\xi_j,c_j),(s_i,\xi_i,c_i),\ldots\rangle \mid a[\vec{s}] \mid \prod_{m\in I} s_m[\mathtt{i}:\vec{h_{im}},\mathtt{o}:\vec{h_{om}}])$$

Then $P_1 \approx P_2$.

Proof. The first step is to consider the recursive definition G.1of the LN transform. We then apply lemmas G.4 and G.7.

The final result gives the equivalence between the simple server and its Lauer-Needham transform.

Theorem G.1. $*a(x).P \mid a[\varepsilon] \approx LN[\![*a(x).P \mid a[\varepsilon]]\!]$

Proof. Since both processes are confluent we can develop a confluent up-to relation along with lemma G.9 to prove bisimulation closure.
Let relation $\mathscr{R}$ such that

$$\mathscr{R} = \{(P_1,P_2),(P_2,P_1) \mid \begin{array}{l} P_1 = *a(x).P \mid R_1 \mid \ldots \mid R_n \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}] \\ R_1,\ldots,R_n \text{ blocking subterms of } P \\ P_2 = \mathsf{Loop} \mid \mathsf{CodeBlocks} \mid r\langle s_j,\ldots,s_{j-1}\rangle \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]\} \end{array}$$

We will prove that $\mathscr{R}$ is a bisimulation up-to confluence. For observable actions bisimulation definition holds trivially since if $P_1 \xrightarrow{\ell} P_1'$ then $P_2 \xrightarrow{\ell} P_2'$ and $P_1'\mathscr{R}P_2'$.
Let $P_2 \longrightarrow Q' \mid \mathsf{CodeBlocks} \mid r\langle s_j,\ldots,s_{j-1}\rangle \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$ then
$P_1 \Longrightarrow *a(x).P \mid R_1 \mid \ldots \mid R_j' \mid \ldots \mid R_n \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$, where $R_j \Longrightarrow R_j'$ and R_j' is a blocking server subterm of P and $Q' \mid \mathsf{CodeBlocks} \mid r\langle s_j,\ldots,s_{j-1}\rangle \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}] \Longrightarrow$ $\mathsf{Loop} \mid \mathsf{CodeBlocks} \mid r\langle s_{j+1},\ldots,s_j\rangle \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$.
For the symmetric case
If $P_1 \longrightarrow *a(x).P \mid R_1 \mid \ldots \mid R_i' \mid \ldots \mid R_n \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$ then we choose a processe $P_2' \approx P_2$ (from lemma G.4) such that
$P_2' = \mathsf{Loop} \mid \mathsf{CodeBlocks} \mid r\langle s_i,s_j\ldots,s_{j+1}\rangle \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$

Now we can observe $P_2' \Longrightarrow \mathsf{Loop} \mid \mathsf{CodeBlocks} \mid r\langle s_j,\ldots,s_i\rangle \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$ and $*a(x).P \mid R_1 \mid \ldots \mid R_i' \mid \ldots \mid R_n \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}] \Longrightarrow *a(x).P \mid R_1 \mid \ldots \mid R_i'' \mid \ldots \mid R_n \mid B_1 \mid \ldots \mid B_n \mid a[\vec{s}]$ where R_i'' is a blocking subterm of P.
This completes the proof.

H Performance Evaluation of the LN-Transform

This Appendix presents a benchmark evaluation of the type-directed transformation from multithreaded to thread-eliminated (i.e. event-driven) programs by the LN-transform (§ 4). The benchmark compares the performance of implementations of the Server processes from Examples 4.1 and 4.1 under load from a varying number of concurrent clients, ranging up to 1000. As discussed earlier, event-driven concurrency is typically regarded as trading performance for more complex implementation; the present benchmark examines these aspects from each side of the trade-off. First, the results show that the LN-transformed Server exhibits the performance and scalability improvements expected of thread-elimination. The second and key point, however, is that the LN-transformation, with formal justification by asynchronous session bisimilarity (Theorem 4.4), ensures the thread-eliminated Server preserves equivalent behaviour to the original. This benchmark thus demonstrates how the session-oriented behavioural equivalence theory presented in this paper can be directly applied to the development of practical tools for real-world concurrent programming.

Benchmark Programs. The implementation and execution of the session-typed benchmark applications uses SJ (Session Java) [13] with extensions for event-driven session programming [12]. The session type implemented by the multithreaded Server (MT) is a slightly modified version of the type from Example 4.1 (to allow variation of the message size), which written in SJ syntax is:

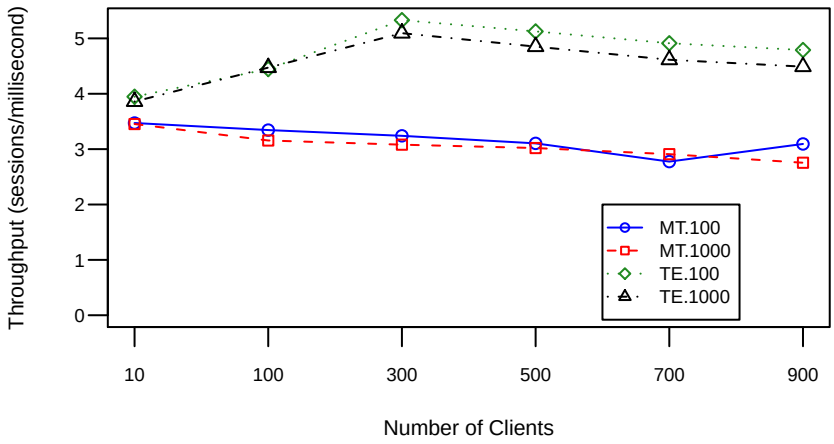

Fig. 13. Throughput of multithreaded (MT) and LN-transformed (TE) SJ Servers under increasing client loads.

```
?(byte[]).!<byte[]>.?(byte[]).!<byte[]>
```

The MT Server spawns a new thread to execute a session of this type for each client that connects. The LN-transformed Server (TE) uses a single-threaded central event-loop to dispatch the input events of the above type across concurrent sessions one-by-one, following Example 4.1. The same Client program, which implements the dual of the above type, is used to interact with both the MT and TE Servers.

Methodology and Environment. The benchmark measures Server throughput in terms of the number of Client sessions completed per millisecond. The benchmark parameters are the message size (100 Bytes and 1 KB), and the number (10, 100, 300, 500, 700 and 900) of concurrent Clients, which simply request and execute repeated sessions with the Server. After the execution of the benchmark configuration has stabilised, a measurement is taken by recording the number of sessions completed by the Server within a 30 s measurement window. For each parameter combination, we take measurements from three windows to each Server instance, and repeat the whole benchmark 20 times.

The benchmark is conducted in the following cluster environment: each node is a Sun Fire x4100 with two 64-bit Opteron 275 processors at 2 GHz and 8 GB of RAM, running 64-bit Mandrakelinux 10.2 (kernel 2.6.11) and connected via gigabit Ethernet. Latency between each node is measured to be 0.5 ms on average (ping 64 bytes). The benchmark applications are compiled and executed using the Sun Java SE compiler and Runtime versions 1.6.0.

Results. Figure 13 gives the mean throughput for the multithreaded (MT) and thread-eliminated (TE) Server implementations for the increasing number of Clients. In all cases, as expected, the TE Server exhibits higher throughput than the MT server. We observe that the throughput of the MT Server decreases steadily due to the thread scheduling overhead. For the TE Server, there is first an increase in throughput, until the saturation point for the Server performance is reached, and then a decline as the number of Clients increases, due to the cost of the event selector handling more Client connections. We also observe higher throughput for the smaller message size: this is because more messages of the smaller size can be transmitted, and thus more sessions can be completed, in the same period of time.